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τ-Equivalences for analysis of concurrent systems modelled by Petri nets with silent transitions*

Igor V. Tarasyuk

Abstract. The paper is devoted to the investigation of behavioural equivalences for Petri nets with silent transitions (τ-equivalences). Basic τ-equivalences and back-forth τ-bisimulation equivalences are supplemented by new ones, giving rise to the complete set of equivalence notions in interleaving / true concurrency and linear / branching time semantics. Their interrelations are examined for the general Petri nets with silent transitions as well as for the subclass of sequential nets. In addition, the preservation of all the equivalence notions by refinements is investigated.

1. Introduction

The notion of equivalence is central in any theory of systems. It allows one to compare systems taking into account particular aspects of their behaviour.

Petri nets became a popular formal model for design of concurrent and distributed systems. Silent transitions are transitions labelled by a special *silent* action τ which represents an internal activity of a system to be modelled and it is invisible for an external observer.

Equivalences abstracting of silent actions are called τ*-equivalences* (these are labelled by the symbol 'τ' to distinguish them of relations not abstracting of silent actions).

Behavioural equivalences can be classified depending on semantics of concurrency they impose. The following kinds of semantics are known.

- *Interleaving*: a concurrent happening of actions is interpreted as their occurrence in any possible order.
- *Step*: a concurrency of actions is a basic notion, but their causal dependencies are not respected.
- *Partial word*: causal dependencies of actions are respected in part via partially ordered multisets (pomsets) of actions, and a pomset may be modelled by a less sequential one (i.e. having less strict partial order).
- *Pomset*: causal dependencies of actions are fully respected, and pomsets of actions should coincide to model each other.

The following basic notions of τ-equivalences are known.

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- *τ-trace equivalences* (they respect only protocols of systems' behaviour): interleaving ($\equiv_i^\tau$) [4], step ($\equiv_s^\tau$) [4], partial word ($\equiv_{pw}^\tau$) [7] and pomset ($\equiv_{pom}^\tau$) [4].
- *Usual τ-bisimulation equivalences* (they respect the branching structure of systems' behaviour): interleaving ($\underline{\leftrightarrow}_i^\tau$) [4], step ($\underline{\leftrightarrow}_s^\tau$) [4], partial word ($\underline{\leftrightarrow}_{pw}^\tau$) [7] and pomset ($\underline{\leftrightarrow}_{pom}^\tau$) [4].
- *ST-τ-bisimulation equivalences* (they respect the duration or maximality of events in systems' behaviour): interleaving ($\underline{\leftrightarrow}_{iST}^\tau$) [7], partial word ($\underline{\leftrightarrow}_{pwST}^\tau$) [7] and pomset ($\underline{\leftrightarrow}_{pomST}^\tau$) [7].
- *History preserving τ-bisimulation equivalences* (they respect the "past" or "history" of systems' behaviour): pomset ($\underline{\leftrightarrow}_{pomh}^\tau$) [4].
- *History preserving ST-τ-bisimulation equivalences* (they respect the "history" and the duration or maximality of events in systems' behaviour): pomset ($\underline{\leftrightarrow}_{pomhST}^\tau$) [4].
- *Usual branching τ-bisimulation equivalences* (they respect the branching structure of systems' behaviour taking a special care of silent actions): interleaving ($\underline{\leftrightarrow}_{ibr}^\tau$) [4].
- *History preserving branching τ-bisimulation equivalences* (they respect the "history" and the branching structure of systems' behaviour taking a special care of silent actions): pomset history preserving ($\underline{\leftrightarrow}_{pomhbr}^\tau$) [4].
- *Isomorphism* ($\simeq$) (i.e. coincidence of systems up to renaming of their components).

Another important group of equivalences are back-forth bisimulation ones based on the idea that a bisimulation relation does not only require systems to simulate behaviour of each other in the forward direction but also when going back in history. They are closely connected with equivalences of logics with past modalities. In [5], the set of all possible back-forth equivalence notions was proposed in interleaving, step, partial word and pomset semantics for event structures without silent actions. The new notion of τ-equivalence was proposed for event structures with silent actions: pomset back pomset forth ($\underline{\leftrightarrow}_{pombpomf}^\tau$) τ-bisimulation equivalence. Its coincidence with $\underline{\leftrightarrow}_{pomhbr}^\tau$ was proved.

Working in the framework of Petri nets with silent transitions, in this paper we continue the research of [6] and extend the set of basic notions of τ-equivalences by interleaving ST-branching τ-bisimulation ($\underline{\leftrightarrow}_{iSTbr}^\tau$) and pomset history preserving ST-branching τ-bisimulation ($\underline{\leftrightarrow}_{pomhSTbr}^\tau$). Thus, we combine a branching idea (taking a special care of silent actions involved in conflicts) and ST-idea (treating occurrences of visible actions as having a beginning and an end). We also define a multi-event structure ($\equiv^{\tau}_{mes}$) equivalence to respect all conflicts of actions. We complete back-forth τ-equivalences from [5] by 6 new notions in interleaving – pomset semantics: interleaving back step forth ($\underline{\leftrightarrow}^{\tau}_{ibsf}$), interleaving back partial word forth ($\underline{\leftrightarrow}^{\tau}_{ibpwf}$), interleaving back pomset forth ($\underline{\leftrightarrow}^{\tau}_{ibpomf}$), step back step forth ($\underline{\leftrightarrow}^{\tau}_{sbsf}$), step back partial word forth ($\underline{\leftrightarrow}^{\tau}_{sbpwf}$) and step back pomset forth ($\underline{\leftrightarrow}^{\tau}_{sbpomf}$) bisimulation equivalences. We compare all the τ-equivalences and obtain a diagram of their interrelations.

In [2], the SM-refinement operator for Petri nets was proposed which "replaces" their transitions by SM-nets, a special subclass of the state machine nets. We check all the τ-equivalences for preservation by SM-refinements to find out which of them may be used in top-down design. We show that $\underline{\leftrightarrow}^{\tau}_{iSTbr}$, $\underline{\leftrightarrow}^{\tau}_{pomhSTbr}$ and $\equiv^{\tau}_{mes}$, i.e., all the new basic equivalences considered in this paper are preserved by SM-refinements. Thus, we have branching and conflict preserving equivalences which may be used for multilevel design. In the literature stability w.r.t. SM-refinements was proved only for $\underline{\leftrightarrow}^{\tau}_{pomhST}$, see [2], and for $\underline{\leftrightarrow}^{\tau}_{iST}$, see [3]. For other ST-$\tau$-bisimulation equivalences, preservation was proved in [7], but it was done on event structures and another refinement operator was used. The preservation of trace τ-equivalences was not established before. Thus, our results for $\underline{\leftrightarrow}^{\tau}_{pwST}$, $\underline{\leftrightarrow}^{\tau}_{pomST}$, $\equiv^{\tau}_{pw}$ and $\equiv^{\tau}_{pom}$ are also new.

In addition, we investigate the interrelations of all the τ-equivalence notions on sequential nets to understand the impact of concurrency on equivalence notions. In the literature, only the coincidence of $\underline{\leftrightarrow}^{\tau}_{i}$ and $\underline{\leftrightarrow}^{\tau}_{pomh}$ was proved on sequential nets, see [2]. We extend these results to all the mentioned equivalence relations.

Due to the space restrictions, we do not present any definitions here and concentrate more on the presentation of the main results obtained. Some basic notions can be found in [6].

2. Interrelations between the τ-equivalences

Let us consider interrelations between all the mentioned τ-equivalences.

Proposition 1. *Let $\star \in \{i, s, pw, pom\}$. For nets N and N' we have:*

1. $N \underline{\leftrightarrow}^{\tau}_{pwb\star f} N' \;\Leftrightarrow\; N \underline{\leftrightarrow}^{\tau}_{pomb\star f} N'$;
2. $N \underline{\leftrightarrow}^{\tau}_{\star bif} N' \;\Leftrightarrow\; N \underline{\leftrightarrow}^{\tau}_{\star b\star f} N'$;
3. $N \underline{\leftrightarrow}^{\tau}_{ibif} N' \;\Leftrightarrow\; N \underline{\leftrightarrow}^{\tau}_{ibr} N'$ *[5]*;
4. $N \underline{\leftrightarrow}^{\tau}_{pombpomf} N' \;\Leftrightarrow\; N \underline{\leftrightarrow}^{\tau}_{pomhbr} N'$ *[5]*;
5. $N \underline{\leftrightarrow}^{\tau}_{iSTbr} N' \;\Rightarrow\; N \underline{\leftrightarrow}^{\tau}_{ibsf} N'$.

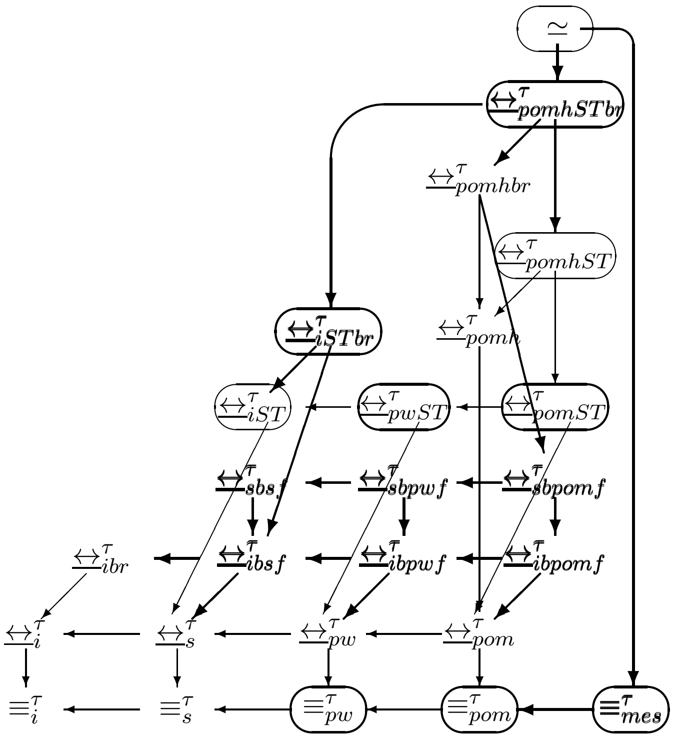

Figure 1. Interrelations between the τ-equivalences and their preservation by SM-refinements

In the following, the symbol '_' will denote an empty alternative. The symbols of equivalences subscribed by it are considered as having no subscriptions.

Theorem 1. *Let* $\leftrightarrow, \twoheadleftrightarrow \in \{\equiv^\tau, \underline{\leftrightarrow}^\tau, \simeq\}$ *and* $\star, \star\star \in \{\_, i, s, pw, pom, iST, pwST, pomST, pomh, pomhST, ibr, pomhbr, iSTbr, pomhSTbr, mes, ibsf, ibpwf, ibpomf, sbsf, sbpwf, sbpomf\}$. *For nets* N *and* N' *we have:* $N \leftrightarrow_\star N' \Rightarrow N \twoheadleftrightarrow_{\star\star} N'$ *iff there exists a directed path from* $\leftrightarrow_\star$ *to* $\twoheadleftrightarrow_{\star\star}$ *in the graph in Figure 1.*

In Figure 1, the new equivalence notions introduced in this paper are printed in bold font, and the newly established interrelations between the equivalences are depicted by thick lines.

3. Transition refinement

In this section we treat the considered τ-equivalences for preservation by SM-transition refinements [2].

A refinement operator $ref(N, a, D)$ "replaces" all transitions of a net N labelled by an action a by SM-net D. SM-nets are a special subclass of state machine nets. An equivalence is *preserved by refinements*, if equivalent nets remain equivalent after applying any refinement operator to them.

Theorem 2. *Let* $\leftrightarrow \in \{\equiv^\tau, \underline{\leftrightarrow}^\tau, \simeq\}$ *and* $\star \in \{\_, i, s, pw, pom, iST, pwST, pomST, pomh, pomhST, ibr, iSTbr, pomhSTbr, pomhbr, mes, ibsf, ibpwf, ibpomf, sbsf, sbpwf, sbpomf\}$. *For nets* N, N' *and an action* a *which labels transitions of these nets and SM-net* D : $N \leftrightarrow_\star N' \Rightarrow ref(N, a, D) \leftrightarrow_\star ref(N', a, D)$ *iff the equivalence* $\leftrightarrow_\star$ *is in oval in Figure 1.*

In Figure 1, thick ovals correspond to the new results on preservation by SM-refinements here obtained.

4. The τ-equivalences on sequential nets

Let us consider the τ-equivalences on sequential nets, where no two transitions can be fired concurrently in any reachable marking.

Proposition 2. *For sequential nets* N *and* N' *we have:*

1. $N \equiv_i^\tau N' \Leftrightarrow N \equiv_{pom}^\tau N'$;
2. $N \underline{\leftrightarrow}_i^\tau N' \Leftrightarrow N \underline{\leftrightarrow}_{pomh}^\tau N'$ [2];
3. $N \underline{\leftrightarrow}_{iST}^\tau N' \Leftrightarrow N \underline{\leftrightarrow}_{pomhST}^\tau N'$;
4. $N \underline{\leftrightarrow}_{ibr}^\tau N' \Leftrightarrow N \underline{\leftrightarrow}_{pomhbr}^\tau N'$;
5. $N \underline{\leftrightarrow}_{iSTbr}^\tau N' \Leftrightarrow N \underline{\leftrightarrow}_{pomhSTbr}^\tau N'$.

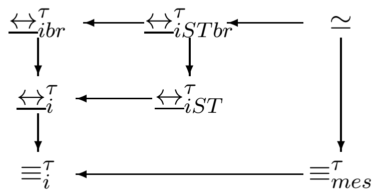

Figure 2. Interrelations between the τ-equivalences on sequential nets

Theorem 3. *Let* $\leftrightarrow, \leftrightarrow\!\!\!\leftrightarrow \in \{\equiv^{\tau}, \underline{\leftrightarrow}^{\tau}, \simeq\}$, $\star, \star\star \in \{\_, i, iST, ibr, iSTbr, mes\}$. *For sequential nets* N *and* N' *we have:* $N \leftrightarrow_{\star} N' \;\Rightarrow\; N \leftrightarrow\!\!\!\leftrightarrow_{\star\star} N'$ *iff there exists a directed path from* $\leftrightarrow_{\star}$ *to* $\leftrightarrow\!\!\!\leftrightarrow_{\star\star}$ *in the graph in Figure 2.*

5. Conclusion

In this paper, a group of basic τ-equivalences and back-forth τ-bisimulation equivalences has been examined and supplemented with new ones. We have also compared them on the whole class of Petri nets as well as on a subclass of sequential nets. All the considered τ-equivalences have been checked for preservation by SM-refinements.

Further research may consist in the investigation of τ-variants of place bisimulation equivalences [1] which are used for effective semantically correct reduction of nets. In this paper a notion of interleaving place τ-bisimulation equivalence has been proposed, and its usefulness for simplification of concurrent systems has been demonstrated.

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