τ-Equivalences and Refinement *

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Abstract

The talk is devoted to investigation of behavioural equivalences for concurrent systems modeled by Petri nets with silent transitions. Basic τ-equivalences and back-forth τ-bisimulation equivalences known from the literature are supplemented by new ones, giving rise to a complete set of the equivalence notions in interleaving / true concurrency and linear / branching time semantics. Their interrelations are examined for the general class of nets as well as for their subclasses of nets without silent transitions and sequential nets (nets without concurrent transitions). In addition, the preservation of all the equivalence notions by refinements (allowing one to consider the systems to be modeled on a lower abstraction levels) is investigated.

Key words & phrases: Petri nets with silent transitions, sequential nets, basic τ-equivalences, back-forth τ-bisimulation equivalences, refinement.

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Labelled nets

Let $Act = \{a, b, \ldots\}$ be a set of *action names* or *labels*. The symbol $\tau \notin Act$ denotes a special *silent* action which represents internal activity of system to be modelled and invisible to external observer. We denote $Act_\tau = Act \cup \{\tau\}$.

Definition 1 *A* labelled net *is a quadruple $N = \langle P_N, T_N, F_N, l_N \rangle$, where:*

- $P_N = \{p, q, \ldots\}$ *is a set of* places*;*
- $T_N = \{t, u, \ldots\}$ *is a set of* transitions*;*
- $F_N : (P_N \times T_N) \cup (T_N \times P_N) \to \mathbf{N}$ *is the* flow relation *with weights (*$\mathbf{N}$ *denotes a set of natural numbers);*
- $l_N : T_N \to Act_\tau$ *is a* labelling *of transitions with action names.*

Given labelled nets $N = \langle P_N, T_N, F_N, l_N \rangle$ and $N' = \langle P_{N'}, T_{N'}, F_{N'}, l_{N'} \rangle$. A mapping $\beta : P_N \cup T_N \to P_{N'} \cup T_{N'}$ is an *isomorphism* between N and N', denoted by $\beta : N \simeq N'$, if:

1. β is a bijection s.t. $\beta(P_N) = P_{N'}$ and $\beta(T_N) = T_{N'}$;
2. $\forall p \in P_N\ \forall t \in T_N\ F_N(p, t) = F_{N'}(\beta(p), \beta(t))$ and $F_N(t, p) = F_{N'}(\beta(t), \beta(p))$;
3. $\forall t \in T_N\ l_N(t) = l_{N'}(\beta(t))$.

Labelled nets N and N' are *isomorphic*, denoted by $N \simeq N'$, if $\exists \beta : N \simeq N'$.

Given a labelled net N and some transition $t \in T_N$, the *precondition* and *postcondition* of t, denoted by ${}^\bullet t$ and $t^\bullet$ respectively, are the multisets defined in such a way: $({}^\bullet t)(p) = F_N(p, t)$ and $(t^\bullet)(p) = F_N(t, p)$. Analogous definitions are introduced for places: $({}^\bullet p)(t) = F_N(t, p)$ and $(p^\bullet)(t) = F_N(p, t)$. Let ${}^\circ N = \{p \in P_N \mid {}^\bullet p = \emptyset\}$ be a set of *initial (input)* places of N and $N^\circ = \{p \in P_N \mid p^\bullet = \emptyset\}$ be a set of *final (output)* places of N.

A labelled net N is *acyclic*, if there exist no transitions $t_0, \ldots, t_n \in T_N$ s.t. $t_{i-1}^\bullet \cap {}^\bullet t_i \neq \emptyset$ $(1 \leq i \leq n)$ and $t_0 = t_n$. A labelled net N is *ordinary* if $\forall p \in P_N$ ${}^\bullet p$ and $p^\bullet$ are proper sets (not multisets).

Let $N = \langle P_N, T_N, F_N, l_N \rangle$ be acyclic ordinary labelled net and $x, y \in P_N \cup T_N$. Let us introduce the following notions.

- $x \prec_N y \ \Leftrightarrow\ x F_N^+ y$, where F_N^+ is a transitive closure of F_N (*strict causal dependence* relation);
- $x \preceq_N y \ \Leftrightarrow\ (x \prec_N y) \vee (x = y)$ (a relation of *causal dependence*);

- $x \#_N y \;\Leftrightarrow\; \exists t, u \in T_N \; (t \neq u,\; {}^\bullet t \cap {}^\bullet u \neq \emptyset,\; t \preceq_N x,\; u \preceq_N y)$ (a relation of *conflict*);
- $\downarrow_N x = \{y \in P_N \cup T_N \mid y \prec_N x\}$ (the set of *strict predecessors* of x).

A set $T \subseteq T_N$ is *left-closed* in N, if $\forall t \in T \; (\downarrow_N t) \cap T_N \subseteq T$.

Marked nets

A *marking* of a labelled net N is a multiset $M \in \mathcal{M}(P_N)$.

Definition 2 *A* marked net (net) *is a tuple* $N = \langle P_N, T_N, F_N, l_N, M_N \rangle$*, where* $\langle P_N, T_N, F_N, l_N \rangle$ *is a labelled net and* $M_N \in \mathcal{M}(P_N)$ *is the* initial *marking.*

Given nets $N = \langle P_N, T_N, F_N, l_N, M_N \rangle$ and $N' = \langle P_{N'}, T_{N'}, F_{N'}, l_{N'}, M_{N'} \rangle$. A mapping $\beta : P_N \cup T_N \to P_{N'} \cup T_{N'}$ is an *isomorphism* between N and N', denoted by $\beta : N \simeq N'$, if:

1. $\beta : \langle P_N, T_N, F_N, l_N \rangle \simeq \langle P_{N'}, T_{N'}, F_{N'}, l_{N'} \rangle$;
2. $\forall p \in P_N \;\; M_N(p) = M_{N'}(\beta(p))$.

Nets N and N' are *isomorphic*, denoted by $N \simeq N'$, if $\exists \beta : N \simeq N'$.

Let $M \in \mathcal{M}(P_N)$ be a marking of a net N. A transition $t \in T_N$ is *fireable* in M, if ${}^\bullet t \subseteq M$. If t is fireable in M, its firing yields a new marking $\widetilde{M} = M - {}^\bullet t + t^\bullet$, denoted by $M \xrightarrow{t} \widetilde{M}$. A marking M of a net N is *reachable*, if $M = M_N$ or there exists a reachable marking $\widehat{M}$ of N s.t. $\widehat{M} \xrightarrow{t} M$ for some $t \in T_N$. $Mark(N)$ denotes a *set of all reachable* markings of a net N.

Partially ordered sets

Definition 3 *A* labelled partially ordered set (lposet) *is a triple* $\rho = \langle X, \prec, l \rangle$, *where:*

- $X = \{x, y, \ldots\}$ *is some set;*
- $\prec \subseteq X \times X$ *is a strict partial order (irreflexive transitive relation) over* X*;*
- $l : X \to Act_\tau$ *is a* labelling *function.*

Let $\rho = \langle X, \prec, l \rangle$ be lposet and $x \in X, Y \subseteq X$. Then $\downarrow x = \{y \in X \mid y \prec x\}$ is a set of *strict predecessors* of x. A *restriction* of ρ to the set Y is defined as follows: $\rho|_Y = \langle Y, \prec \cap (Y \times Y), l|_Y \rangle$.

Let $\rho = \langle X, \prec, l \rangle$ and $\rho' = \langle X', \prec', l' \rangle$ be lposets.

A mapping $\beta : X \to X'$ is a *label-preserving bijection* between ρ and ρ', denoted by $\beta : \rho \asymp \rho'$, if:

1. β is a bijection;
2. $\forall x \in X \ l(x) = l'(\beta(x))$.

We write $\rho \asymp \rho'$, if $\exists \beta : \rho \asymp \rho'$.

A mapping $\beta : X \to X'$ is a *homomorphism* between ρ and ρ', denoted by $\beta : \rho \sqsubseteq \rho'$, if:

1. $\beta : \rho \asymp \rho'$;
2. $\forall x, y \in X \ x \prec y \ \Rightarrow \ \beta(x) \prec' \beta(y)$.

We write $\rho \sqsubseteq \rho'$, if $\exists \beta : \rho \sqsubseteq \rho'$.

A mapping $\beta : X \to X'$ is an *isomorphism* between ρ and ρ', denoted by $\beta : \rho \simeq \rho'$, if $\beta : \rho \sqsubseteq \rho'$ and $\beta^{-1} : \rho' \sqsubseteq \rho$. Lposets ρ and ρ' are *isomorphic*, denoted by $\rho \simeq \rho'$, if $\exists \beta : \rho \simeq \rho'$.

Definition 4 Partially ordered multiset (pomset) *is an isomorphism class of lposets.*

Event structures

Definition 5 *A* labelled event structure (LES) *is a quadruple* $\xi = \langle X, \prec, \#, l \rangle$, *where:*

- $X = \{x, y, \ldots\}$ *is a set of* events*;*
- $\prec \subseteq X \times X$ *is a strict partial order, a* causal dependence *relation, which satisfies to the principle of* finite causes*:* $\forall x \in X \ | \downarrow x| < \infty$*;*
- $\# \subseteq X \times X$ *is an irreflexive symmetrical* conflict relation*, which satisfies to the principle of* conflict heredity*:* $\forall x, y, z \in X \ x\#y \prec z \ \Rightarrow \ x\#z$*;*
- $l : X \to Act_\tau$ *is a* labelling *function.*

Let $\xi = \langle X, \prec, \#, l \rangle$ be LES and $Y \subseteq X$. A *restriction* of ξ to the set Y is defined as follows: $\xi|_Y = \langle Y, \prec \cap (Y \times Y), \# \cap (Y \times Y), l|_Y \rangle$.

Let $\xi = \langle X, \prec, \#, l \rangle$ and $\xi' = \langle X', \prec', \#', l' \rangle$ be LES's. A mapping $\beta : X \to X'$ is an *isomorphism* between ξ and ξ', denoted by $\beta : \xi \simeq \xi'$, if:

1. β is a bijection;
2. $\forall x \in X \ l(x) = l'(\beta(x))$;
3. $\forall x, y \in X \ x \prec y \ \Leftrightarrow \ \beta(x) \prec' \beta(y)$;
4. $\forall x, y \in X \ x\#y \ \Leftrightarrow \ \beta(x)\#'\beta(y)$.

LES's ξ and ξ' are *isomorphic*, denoted by $\xi \simeq \xi'$, if $\exists \beta : \xi \simeq \xi'$.

Definition 6 *A* multi-event structure (MES) *is an isomorphism class of LES's.*

C-processes

Definition 7 *A* causal net *is an acyclic ordinary labelled net* $C = \langle P_C, T_C, F_C, l_C \rangle$*, s.t.:*

1. $\forall r \in P_C$ $|{}^\bullet r| \leq 1$ *and* $|r^\bullet| \leq 1$*, i.e. places are unbranched;*
2. $\forall x \in P_C \cap T_C$ $|\downarrow_C x| < \infty$*, i.e. a set of causes is finite.*

Let us note that on the basis of any causal net $C = \langle P_C, T_C, F_C, l_C \rangle$ one can define lposet $\rho_C = \langle T_C, \prec_N \cap (T_C \times T_C), l_C \rangle$.

The fundamental property of causal nets is: if C is a causal net, then there exists a sequence of transition firings ${}^\circ C = L_0 \xrightarrow{v_1} \cdots \xrightarrow{v_n} L_n = C^\circ$ s.t. $L_i \subseteq P_C$ $(0 \leq i \leq n)$, $P_C = \cup_{i=0}^{n} L_i$ and $T_C = \{v_1, \ldots, v_n\}$. Such a sequence is called a *full execution* of C.

Definition 8 *Given a net N and a causal net C. A mapping* $\varphi : P_C \cup T_C \to P_N \cup T_N$ *is an* embedding *of C into N, denoted by* $\varphi : C \to N$*, if:*

1. $\varphi(P_C) \in \mathcal{M}(P_N)$ *and* $\varphi(T_C) \in \mathcal{M}(T_N)$*, i.e. sorts are preserved;*
2. $\forall v \in T_C$ ${}^\bullet\varphi(v) = \varphi({}^\bullet v)$ *and* $\varphi(v)^\bullet = \varphi(v^\bullet)$*, i.e. flow relation is respected;*
3. $\forall v \in T_C$ $l_C(v) = l_N(\varphi(v))$*, i.e. labelling is preserved.*

Since embeddings respect the flow relation, if ${}^\circ C \xrightarrow{v_1} \cdots \xrightarrow{v_n} C^\circ$ is a full execution of C, then $M = \varphi({}^\circ C) \xrightarrow{\varphi(v_1)} \cdots \xrightarrow{\varphi(v_n)} \varphi(C^\circ) = \widetilde{M}$ is a sequence of transition firings in N.

Definition 9 *A* fireable in marking M C-process (process) *of a net N is a pair* $\pi = (C, \varphi)$*, where C is a causal net and* $\varphi : C \to N$ *is an embedding s.t.* $M = \varphi({}^\circ C)$*. A fireable in M_N process is a* process *of N.*

We write $\Pi(N, M)$ for a *set of all fireable* in marking M processes of a net N and $\Pi(N)$ for the *set of all processes* of a net N. The *initial* process of a net N is $\pi_N = (C_N, \varphi_N) \in \Pi(N)$, s.t. $T_{C_N} = \emptyset$. If $\pi \in \Pi(N, M)$, then firing of this process transforms a marking M into $\widetilde{M} = M - \varphi({}^\circ C) + \varphi(C^\circ) = \varphi(C^\circ)$, denoted by $M \xrightarrow{\pi} \widetilde{M}$.

Let $\pi = (C, \varphi)$, $\tilde{\pi} = (\widetilde{C}, \tilde{\varphi}) \in \Pi(N)$, $\hat{\pi} = (\widehat{C}, \hat{\varphi}) \in \Pi(N, \varphi(C^\circ))$. A process π is a *prefix* of a process $\tilde{\pi}$, if $T_C \subseteq T_{\widetilde{C}}$ is a left-closed set in $\widetilde{C}$. A process $\hat{\pi}$ is a *suffix* of a process $\tilde{\pi}$, if $T_{\widehat{C}} = T_{\widetilde{C}} \setminus T_C$. In such a case a process $\tilde{\pi}$ is an *extension* of π *by process* $\hat{\pi}$, and $\hat{\pi}$ is an *extending* process for π, denoted by $\pi \xrightarrow{\hat{\pi}} \tilde{\pi}$. We write $\pi \to \tilde{\pi}$, if $\pi \xrightarrow{\hat{\pi}} \tilde{\pi}$ for some $\hat{\pi}$.

A process $\tilde{\pi}$ is an extension of a process π *by one transition*, denoted by $\pi \xrightarrow{v} \tilde{\pi}$ or $\pi \xrightarrow{a} \tilde{\pi}$, if $\pi \xrightarrow{\hat{\pi}} \tilde{\pi}$, $T_{\widehat{C}} = \{v\}$ and $l_{\widehat{C}}(v) = a$.

A process $\tilde{\pi}$ is an extension of a process π *by sequence of transitions*, denoted by $\pi \xrightarrow{\sigma} \tilde{\pi}$ or $\pi \xrightarrow{\omega} \tilde{\pi}$, if $\exists \pi_i \in \Pi(N)$ $(1 \leq i \leq n)$ $\pi \xrightarrow{v_1} \pi_1 \xrightarrow{v_2} \ldots \xrightarrow{v_n} \pi_n = \tilde{\pi}$, $\sigma = v_1 \cdots v_n$ and $l_{\widehat{C}}(\sigma) = \omega$.

A process $\tilde{\pi}$ is an extension of a process π *by multiset of transitions*, denoted by $\pi \xrightarrow{V} \tilde{\pi}$ or $\pi \xrightarrow{A} \tilde{\pi}$, if $\pi \xrightarrow{\hat{\pi}} \tilde{\pi}$, $\prec_{\widehat{C}} = \emptyset$, $T_{\widehat{C}} = V$ and $l_{\widehat{C}}(V) = A$.

O-processes

Definition 10 *An* occurrence net *is an acyclic ordinary labelled net* $O = \langle P_O, T_O, F_O, l_O \rangle$*, s.t.:*

1. $\forall r \in P_O \ |{}^\bullet r| \le 1$*, i.e. there are no backwards conflicts;*
2. $\forall x \in P_O \cup T_O \ \neg(x \#_O x)$*, i.e. conflict relation is irreflexive;*
3. $\forall x \in P_O \cup T_O \ |\downarrow_O x| < \infty$*, i.e. set of causes is finite.*

Let $O = \langle P_O, T_O, F_O, l_O \rangle$ be occurrence net and $N = \langle P_N, T_N, F_N, l_N, M_N \rangle$ be some net. A mapping $\psi : P_O \cup T_O \to P_N \cup T_N$ is an *embedding* O into N, notation $\psi : O \to N$, if:

1. $\psi(P_O) \in \mathcal{M}(P_N)$ and $\psi(T_O) \in \mathcal{M}(T_N)$. i.e. sorts are preserved;
2. $\forall v \in T_O \ l_O(v) = l_N(\psi(v))$, i.e. labelling is preserved;
3. $\forall v \in T_O \ {}^\bullet\psi(v) = \psi({}^\bullet v)$ and $\psi(v)^\bullet = \psi(v^\bullet)$, i.e. flow relation is respected;
4. $\forall v, w \in T_O \ ({}^\bullet v = {}^\bullet w) \wedge (\psi(v) = \psi(w)) \Rightarrow \ v = w$, i.e. there are no "superfluous" conflicts.

Let us note that on the basis of any occurrence net O one can define LES $\xi_O = \langle T_O, \prec_O \cap (T_O \times T_O), \#_O \cap (T_O \times T_O), l_O \rangle$.

Definition 11 *An* O-process *of a net* N *is a pair* $\varpi = (O, \psi)$*, where* O *is an occurrence net and* $\psi : O \to N$ *is an embedding s.t.* $M_N = \psi({}^\circ O)$*.*

We write $\wp(N)$ for a set of *all O-processes* of a net N. The *initial* O-process of a net N coincides with its initial C-process, i.e. $\varpi_N = \pi_N$.

Let $\varpi = (O, \psi)$, $\tilde{\varpi} = (\widetilde{O}, \tilde{\psi}) \in \wp(N)$, $O = \langle P_O, T_O, F_O, l_O \rangle$, $\widetilde{O} = \langle P_{\widetilde{O}}, T_{\widetilde{O}}, F_{\widetilde{O}}, l_{\widetilde{O}} \rangle$. An O-process ϖ is a *prefix* of a process $\tilde{\varpi}$, if $T_O \subseteq T_{\widetilde{O}}$ is a left-closed set in $\widetilde{O}$. In such a case O-process $\tilde{\varpi}$ is an *extension* of ϖ, and $\hat{\varpi}$ is an *extending* O-process for ϖ, denoted by $\varpi \to \tilde{\varpi}$.

An O-process ϖ of a net N is *maximal*, if it cannot be extended, i.e. $\forall \varpi = (O, \psi)$ s.t. $\varpi \to \tilde{\varpi} : \ T_{\widetilde{O}} \setminus T_O = \emptyset$. A set of all maximal O-processes of a net N consists of the unique (up to isomorphism) O-process $\varpi_{max} = (O_{max}, \psi_{max})$. In such a case an isomorphism class of occurrence net O_{max} is an *unfolding* of a net N, notation $\mathcal{U}(N)$. On the basis of unfolding $\mathcal{U}(N)$ of a net N one can define MES $\mathcal{E}(N) = \xi_{\mathcal{U}(N)}$ which is an isomorphism class of LES ξ_O for $O \in \mathcal{U}(N)$.

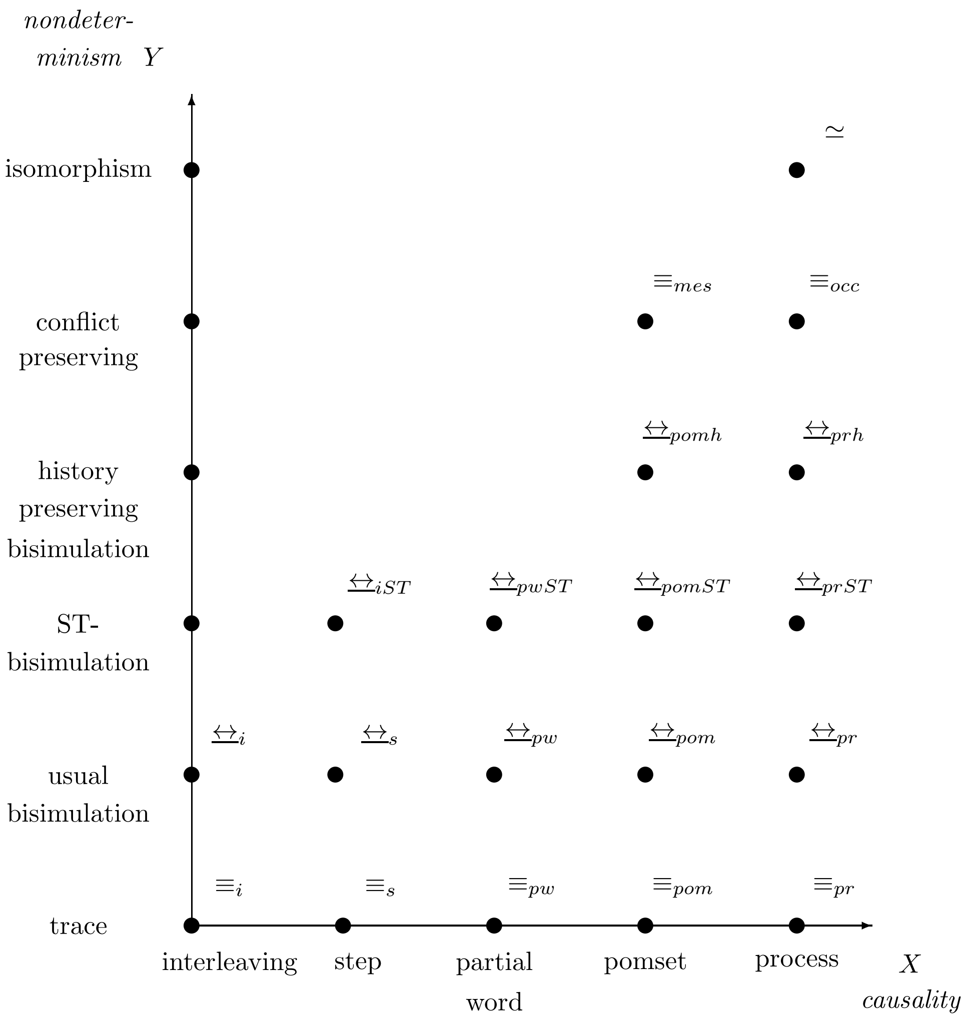

Figure 1: Classification of basic equivalences

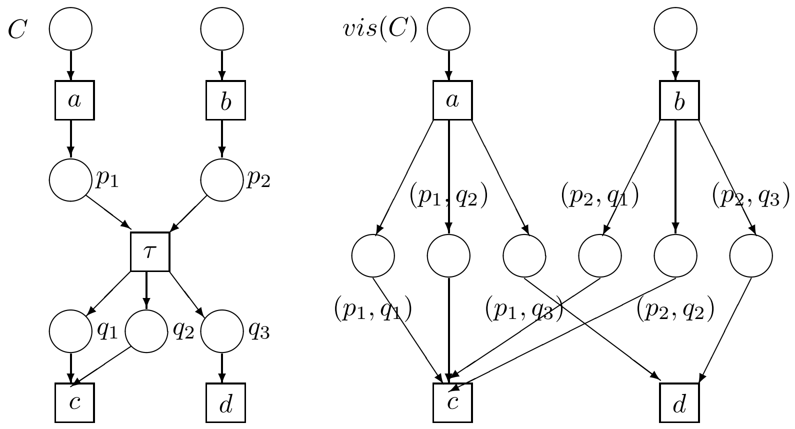

Figure 2: An application of the mapping vis to a causal net

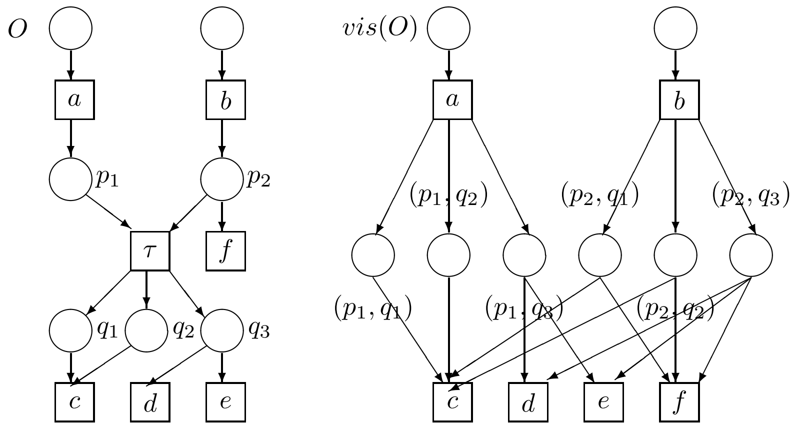

Figure 3: An application of the mapping vis to an occurrence net

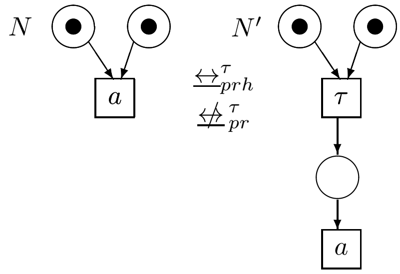

Figure 4: A crash of interrelations of the process τ-bisimulation equivalences comparing with that of the process bisimulation equivalences

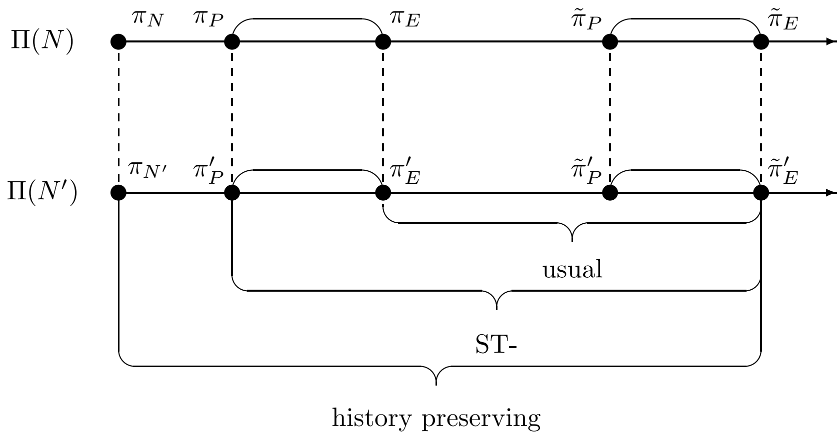

Figure 5: A distinguish ability of the bisimulation equivalences

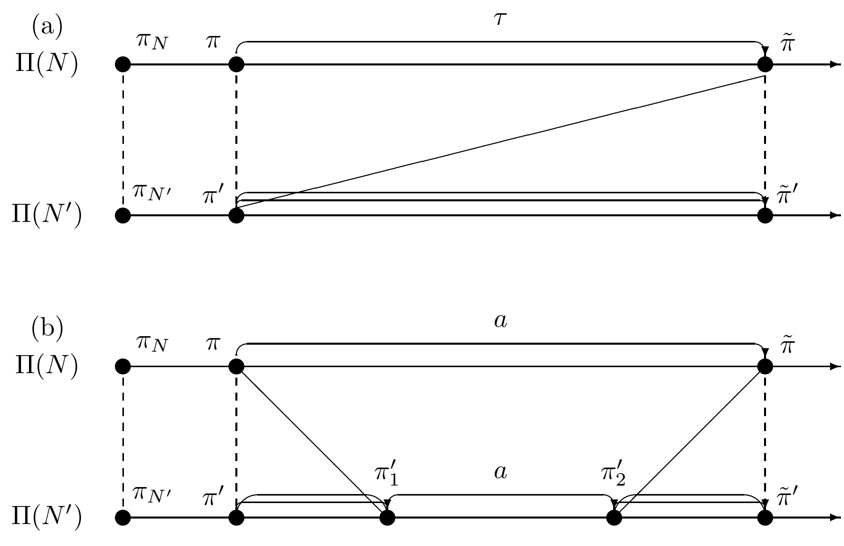

Figure 6: A distinguish ability of the usual and the branching τ-bisimulation equivalences

τ-trace equivalences

We denote the empty string by the symbol ε.

Let $\sigma = a_1 \cdots a_n \in Act_\tau^*$. We define $vis(\sigma)$ as follows (in the following definition $a \in Act_\tau$).

1. $vis(\varepsilon) = \varepsilon$;
2. $vis(\sigma a) = \begin{cases} vis(\sigma)a, & a \neq \tau; \\ vis(\sigma), & a = \tau. \end{cases}$

Definition 12 *A* visible interleaving trace *of a net N is a sequence $vis(a_1 \cdots a_n) \in Act^*$ s.t. $\pi_N \stackrel{a_1}{\to} \pi_1 \stackrel{a_2}{\to} \ldots \stackrel{a_n}{\to} \pi_n$, where π_N is the initial process of a net N and $\pi_i \in \Pi(N)$ $(1 \leq i \leq n)$. We denote a set of* all visible interleaving traces *of a net N by $VisIntTraces(N)$. Two nets N and N' are* interleaving τ-trace equivalent, *denoted by $N \equiv_i^\tau N'$, if $VisIntTraces(N) = VisIntTraces(N')$.*

Let $\Sigma = A_1 \cdots A_n \in (\mathcal{M}(Act_\tau))^*$. We define $vis(\Sigma)$ as follows (in the following definition $A \in \mathcal{M}(Act_\tau)$).

1. $vis(\varepsilon) = \varepsilon$;
2. $vis(\Sigma A) = \begin{cases} vis(\Sigma)(A \cap Act), & A \cap Act \neq \emptyset; \\ vis(\Sigma), & \text{otherwise}. \end{cases}$

Definition 13 *A* visible step trace *of a net N is a sequence $vis(A_1 \cdots A_n) \in (\mathcal{M}(Act))^*$ s.t. $\pi_N \stackrel{A_1}{\to} \pi_1 \stackrel{A_2}{\to} \ldots \stackrel{A_n}{\to} \pi_n$, where π_N is the initial process of a net N and $\pi_i \in \Pi(N)$ $(1 \leq i \leq n)$. We denote a set of* all visible step traces *of a net N by $VisStepTraces(N)$. Two nets N and N' are* step τ-trace equivalent, *denoted by $N \equiv_s^\tau N'$, if $VisStepTraces(N) = VisStepTraces(N')$.*

Let $\rho = \langle X, \prec, l\rangle$ is lposet s.t. $l : X \to Act_\tau$. We denote $vis(X) = \{x \in X \mid l(x) \in Act\}$ and $vis(\rho) = \rho|_{vis(X)}$.

Definition 14 *A* visible pomset trace *of a net N is a pomset $vis(\rho)$, an isomorphism class of lposet $vis(\rho_C)$ for $\pi = (C, \varphi) \in \Pi(N)$. We denote a set of* all visible pomsets *of a net N by $VisPomsets(N)$. Two nets N and N' are* partial word τ-trace equivalent, *denoted by $N \equiv_{pw}^\tau N'$, if $VisPomsets(N) \sqsubseteq VisPomsets(N')$ and $VisPomsets(N') \sqsubseteq VisPomsets(N)$.*

Definition 15 *Two nets N and N' are* pomset τ-trace equivalent, *denoted by $N \equiv_{pom}^\tau N'$, if $VisPomsets(N) = VisPomsets(N')$.*

Usual τ-bisimulation equivalences

Let $C = \langle P_C, T_C, F_C, l_C \rangle$ be C-net. We denote $vis(T_C) = \{v \in T_C \mid l_C(v) \in Act\}$ and $vis(\prec_C) = \prec_C \cap (vis(T_C) \times vis(T_C))$.

Definition 16 *Let N and N' be some nets. A relation $\mathcal{R} \subseteq \Pi(N) \times \Pi(N')$ is a $\star$-τ-*bisimulation *between N and N', $\star \in$*{interleaving, step, partial word, pomset}*, denoted by $\mathcal{R} : N \underline{\leftrightarrow}^{\tau}_{\star} N'$, $\star \in \{i, s, pw, pom\}$, if:*

1. $(\pi_N, \pi_{N'}) \in \mathcal{R}$.
2. $(\pi, \pi') \in \mathcal{R}$, $\pi \stackrel{\hat{\pi}}{\rightarrow} \tilde{\pi}$,

   (a) $|vis(T_{\widehat{C}})| = 1$, *if* $\star = i$;

   (b) $vis(\prec_{\widehat{C}}) = \emptyset$, *if* $\star = s$;

   $\Rightarrow\ \exists \tilde{\pi}' :\ \pi' \stackrel{\hat{\pi}'}{\rightarrow} \tilde{\pi}'$, $(\tilde{\pi}, \tilde{\pi}') \in \mathcal{R}$ *and*

   (a) $vis(\rho_{\widehat{C}'}) \sqsubseteq vis(\rho_{\widehat{C}})$, *if* $\star = pw$;

   (b) $vis(\rho_{\widehat{C}}) \simeq vis(\rho_{\widehat{C}'})$, *if* $\star \in \{i, s, pom\}$.
3. *As item 2, but the roles of N and N' are reversed.*

*Two nets N and N' are $\star$-τ-*bisimulation equivalent*, $\star \in$*{interleaving, step, partial word, pomset}*, denoted by $N \underline{\leftrightarrow}^{\tau}_{\star} N'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}^{\tau}_{\star} N'$, $\star \in \{i, s, pw, pom\}$.*

ST-τ-bisimulation equivalences

Definition 17 *An* ST-τ-process *of a net N is a pair (π_E, π_P) s.t. $\pi_E, \pi_P \in \Pi(N)$, $\pi_P \xrightarrow{\pi_W} \pi_E$ and $\forall v, w \in T_{C_E}$ $(v \prec_{C_E} w) \vee (l_{C_E}(v) = \tau) \;\Rightarrow\; v \in T_{C_P}$.*

In such a case π_E is a process which began working, π_P corresponds to the completed part of π_E, and π_W — to the still working part. Obviously, $\prec_{C_W} = \emptyset$. We denote a set of *all ST-τ-processes* of a net N by $ST^\tau - \Pi(N)$. (π_N, π_N) is the *initial ST-τ-process* of a net N. Let (π_E, π_P), $(\tilde{\pi}_E, \tilde{\pi}_P) \in ST^\tau - \Pi(N)$. We write $(\pi_E, \pi_P) \to (\tilde{\pi}_E, \tilde{\pi}_P)$, if $\pi_E \to \tilde{\pi}_E$ and $\pi_P \to \tilde{\pi}_P$.

Definition 18 *Let N and N' be some nets. A relation $\mathcal{R} \subseteq ST^\tau - \Pi(N) \times ST^\tau - \Pi(N') \times \mathcal{B}$, where $\mathcal{B} = \{\beta \mid \beta : vis(T_C) \to vis(T_{C'}),\ \pi = (C, \varphi) \in \Pi(N),\ \pi' = (C', \varphi') \in \Pi(N')\}$ is a $\star$-*ST-τ-bisimulation *between N and N', $\star \in$*{interleaving, partial word, pomset}*, denoted by $\mathcal{R} : N \underline{\leftrightarrow}^\tau_{\star ST} N'$, $\star \in \{i, pw, pom\}$, if:*

1. *$((\pi_N, \pi_N), (\pi_{N'}, \pi_{N'}), \emptyset) \in \mathcal{R}$.*
2. *$((\pi_E, \pi_P), (\pi'_E, \pi'_P), \beta) \in \mathcal{R} \;\Rightarrow\; \beta : vis(\rho_{C_E}) \asymp vis(\rho_{C'_E})$ and $\beta(vis(T_{C_P})) = vis(T_{C'_P})$.*
3. *$((\pi_E, \pi_P), (\pi'_E, \pi'_P), \beta) \in \mathcal{R}$, $(\pi_E, \pi_P) \to (\tilde{\pi}_E, \tilde{\pi}_P) \;\Rightarrow\; \exists \tilde{\beta}$, $(\tilde{\pi}'_E, \tilde{\pi}'_P) : (\pi'_E, \pi'_P) \to (\tilde{\pi}'_E, \tilde{\pi}'_P)$, $\tilde{\beta}|_{vis(T_{C_E})} = \beta$, $((\tilde{\pi}_E, \tilde{\pi}_P), (\tilde{\pi}'_E, \tilde{\pi}'_P), \tilde{\beta}) \in \mathcal{R}$, and if $\pi_P \xrightarrow{\pi} \tilde{\pi}_E$, $\pi'_P \xrightarrow{\pi'} \tilde{\pi}'_E$, $\gamma = \tilde{\beta}|_{vis(T_C)}$, then:*
   (a) $\gamma^{-1} : vis(\rho_{C'}) \sqsubseteq vis(\rho_C)$, if $\star = pw$;
   (b) $\gamma : vis(\rho_C) \simeq vis(\rho_{C'})$, if $\star = pom$.
4. *As item 3, but the roles of N and N' are reversed.*

Two nets N and N' are $\star$-ST-τ-bisimulation equivalent*, $\star \in$*{interleaving, partial word, pomset}*, denoted by $N \underline{\leftrightarrow}^\tau_{\star ST} N'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}^\tau_{\star ST} N'$, $\star \in \{i, pw, pom\}$.*

History preserving τ-bisimulation equivalences

Definition 19 *Let N and N' be some nets. A relation $\mathcal{R} \subseteq \Pi(N) \times \Pi(N') \times \mathcal{B}$, where $\mathcal{B} = \{\beta \mid \beta : vis(T_C) \to vis(T_{C'}),\ \pi = (C, \varphi) \in \Pi(N),\ \pi' = (C', \varphi') \in \Pi(N')\}$, is a* pomset history preserving τ-bisimulation *between N and N', denoted by $\mathcal{R} : N \underline{\leftrightarrow}^{\tau}_{pomh} N'$, if:*

1. $(\pi_N, \pi_{N'}, \emptyset) \in \mathcal{R}$.
2. $(\pi, \pi', \beta) \in \mathcal{R} \ \Rightarrow\ \beta : vis(\rho_C) \simeq vis(\rho_{C'})$.
3. $(\pi, \pi', \beta) \in \mathcal{R},\ \pi \to \tilde{\pi} \ \Rightarrow\ \exists \tilde{\beta},\ \tilde{\pi}' : \pi' \to \tilde{\pi}',\ \tilde{\beta}|_{vis(T_C)} = \beta,\ (\tilde{\pi}, \tilde{\pi}', \tilde{\beta}) \in \mathcal{R}$.
4. *As item 3, but the roles of N and N' are reversed.*

Two nets N and N' are pomset history preserving τ-bisimulation equivalent*, denoted by $N \underline{\leftrightarrow}^{\tau}_{pomh} N'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}^{\tau}_{pomh} N'$.*

History preserving ST-τ-bisimulation equivalences

Definition 20 *Let N and N' be some nets. A relation $\mathcal{R} \subseteq ST^{\tau}-\Pi(N) \times ST^{\tau}-\Pi(N') \times \mathcal{B}$, where $\mathcal{B} = \{\beta \mid \beta : vis(T_C) \to vis(T_{C'}),\ \pi = (C, \varphi) \in \Pi(N),\ \pi' = (C', \varphi') \in \Pi(N')\}$, is a* pomset history preserving ST-τ-bisimulation *between N and N', denoted by $\mathcal{R} : N \underline{\leftrightarrow}^{\tau}_{pomhST} N'$, if:*

1. $((\pi_N, \pi_N), (\pi_{N'}, \pi_{N'}), \emptyset) \in \mathcal{R}$.
2. $((\pi_E, \pi_P), (\pi'_E, \pi'_P), \beta) \in \mathcal{R} \ \Rightarrow\ \beta : vis(\rho_{C_E}) \simeq vis(\rho_{C'_E})$ *and* $\beta(vis(T_{C_P})) = vis(T_{C'_P})$.
3. $((\pi_E, \pi_P), (\pi'_E, \pi'_P), \beta) \in \mathcal{R},\ (\pi_E, \pi_P) \to (\tilde{\pi}_E, \tilde{\pi}_P) \quad \Rightarrow \quad \exists \tilde{\beta},\ (\tilde{\pi}'_E, \tilde{\pi}'_P) : (\pi'_E, \pi'_P) \to (\tilde{\pi}'_E, \tilde{\pi}'_P),\ \tilde{\beta}|_{vis(T_{C_E})} = \beta,\ ((\tilde{\pi}_E, \tilde{\pi}_P), (\tilde{\pi}'_E, \tilde{\pi}'_P), \tilde{\beta}) \in \mathcal{R}$.
4. *As item 3, but the roles of N and N' are reversed.*

Two nets N and N' are pomset history preserving ST-τ-bisimulation equivalent*, denoted by $N \underline{\leftrightarrow}^{\tau}_{pomhST} N'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}^{\tau}_{pomhST} N'$.*

Usual branching τ-bisimulation equivalences

For some net N and $\pi, \tilde{\pi} \in \Pi(N)$ we write $\pi \Rightarrow \tilde{\pi}$ when $\exists \hat{\pi} = (\widehat{C}, \hat{\varphi})$ s.t. $\pi \xrightarrow{\hat{\pi}} \tilde{\pi}$ and $vis(T_{\widehat{C}}) = \emptyset$.

Definition 21 *Let N and N' be some nets. A relation $\mathcal{R} \subseteq \Pi(N) \times \Pi(N')$ is an* interleaving branching τ-bisimulation *between N and N', denoted by $N \underline{\leftrightarrow}^{\tau}_{ibr} N'$, if:*

1. $(\pi_N, \pi_{N'}) \in \mathcal{R}$.
2. $(\pi, \pi') \in \mathcal{R},\ \pi \xrightarrow{a} \tilde{\pi}\ \Rightarrow$
   - *(a)* $a = \tau$ *and* $(\tilde{\pi}, \pi') \in \mathcal{R}$ *or*
   - *(b)* $a \neq \tau$ *and* $\exists \bar{\pi}',\ \tilde{\pi}' : \pi' \Rightarrow \bar{\pi}' \xrightarrow{a} \tilde{\pi}',\ (\pi, \bar{\pi}') \in \mathcal{R},\ (\tilde{\pi}, \tilde{\pi}') \in \mathcal{R}$.
3. *As item 2, but the roles of N and N' are reversed.*

Two nets N and N' are interleaving branching τ-bisimulation equivalent*, denoted by $N \underline{\leftrightarrow}^{\tau}_{ibr} N'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}^{\tau}_{ibr} N'$.*

History preserving branching τ-bisimulation equivalences

Definition 22 *Let N and N' be some nets. A relation $\mathcal{R} \subseteq \Pi(N) \times \Pi(N') \times \mathcal{B}$, where $\mathcal{B} = \{\beta \mid \beta : T_C \to T_{C'},\ \pi = (C, \varphi) \in \Pi(N),\ \pi' = (C', \varphi') \in \Pi(N')\}$, is a* pomset history preserving branching τ-bisimulation *between N and N', denoted by $N \underline{\leftrightarrow}^{\tau}_{pomhbr} N'$, if:*

1. $(\pi_N, \pi_{N'}, \emptyset) \in \mathcal{R}$.
2. $(\pi, \pi', \beta) \in \mathcal{R} \ \Rightarrow\ \beta : vis(\rho_C) \simeq vis(\rho_{C'})$.
3. $(\pi, \pi', \beta) \in \mathcal{R},\ \pi \to \tilde{\pi}\ \Rightarrow$
   (a) $(\tilde{\pi}, \pi', \beta) \in \mathcal{R}$ *or*
   (b) $\exists \tilde{\beta},\ \bar{\pi}',\ \tilde{\pi}' : \pi' \Rightarrow \bar{\pi}' \to \tilde{\pi}',\ \tilde{\beta}|_{vis(T_C)} = \beta,\ (\pi, \bar{\pi}', \beta) \in \mathcal{R},\ (\tilde{\pi}, \tilde{\pi}', \tilde{\beta}) \in \mathcal{R}$.
4. *As item 3, but the roles of N and N' are reversed.*

Two nets N and N' are pomset history preserving branching τ-bisimulation equivalent*, denoted by $N \underline{\leftrightarrow}^{\tau}_{pomhbr} N'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}^{\tau}_{pomhbr} N'$.*

ST-branching τ-bisimulation equivalences

Let $(\pi_E, \pi_P),\ (\tilde{\pi}_E, \tilde{\pi}_P) \in ST^\tau - \Pi(N)$. We write $(\pi_E, \pi_P) \Rightarrow (\tilde{\pi}_E, \tilde{\pi}_P)$, if $\pi_E \Rightarrow \tilde{\pi}_E$ and $\pi_P \Rightarrow \tilde{\pi}_P$.

Definition 23 *Let N and N' be some nets. A relation $\mathcal{R} \subseteq ST^\tau - \Pi(N) \times ST^\tau - \Pi(N') \times \mathcal{B}$, where $\mathcal{B} = \{\beta \mid \beta : vis(T_C) \to vis(T_{C'}),\ \pi = (C, \varphi) \in \Pi(N),\ \pi' = (C', \varphi') \in \Pi(N')\}$ is an* interleaving ST-branching τ-bisimulation *between N and N', denoted by $\mathcal{R} : N \underline{\leftrightarrow}^\tau_{iSTbr} N'$, if:*

1. $((\pi_N, \pi_N), (\pi_{N'}, \pi_{N'}), \emptyset) \in \mathcal{R}$.
2. $((\pi_E, \pi_P), (\pi'_E, \pi'_P), \beta) \in \mathcal{R} \ \Rightarrow\ \beta : vis(\rho_{C_E}) \asymp vis(\rho_{C'_E})$ *and* $\beta(vis(T_{C_P})) = vis(T_{C'_P})$.
3. $((\pi_E, \pi_P), (\pi'_E, \pi'_P), \beta) \in \mathcal{R},\ (\pi_E, \pi_P) \to (\tilde{\pi}_E, \tilde{\pi}_P) \ \Rightarrow$
   - *(a)* $((\tilde{\pi}_E, \tilde{\pi}_P), (\pi'_E, \pi'_P), \beta) \in \mathcal{R}$ *or*
   - *(b)* $\exists \tilde{\beta},\ (\bar{\pi}'_E, \bar{\pi}'_P),\ (\tilde{\pi}'_E, \tilde{\pi}'_P) :\ (\pi'_E, \pi'_P) \Rightarrow (\bar{\pi}'_E, \bar{\pi}'_P) \to (\tilde{\pi}'_E, \tilde{\pi}'_P),\ \tilde{\beta}|_{vis(T_{C_E})} = \beta,\ ((\pi_E, \pi_P), (\bar{\pi}'_E, \bar{\pi}'_P), \beta) \in \mathcal{R},\ ((\tilde{\pi}_E, \tilde{\pi}_P), (\tilde{\pi}'_E, \tilde{\pi}'_P), \tilde{\beta}) \in \mathcal{R}$.
4. *As item 3, but the roles of N and N' are reversed.*

Two nets N and N' are interleaving ST-branching τ-bisimulation equivalent*, denoted by $N \underline{\leftrightarrow}^\tau_{iSTbr} N'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}^\tau_{iSTbr} N'$.*

History preserving ST-branching τ-bisimulation equivalences

Definition 24 *Let N and N' be some nets. A relation $\mathcal{R} \subseteq ST^\tau - \Pi(N) \times ST^\tau - \Pi(N') \times \mathcal{B}$, where $\mathcal{B} = \{\beta \mid \beta : vis(T_C) \to vis(T_{C'}),\ \pi = (C, \varphi) \in \Pi(N),\ \pi' = (C', \varphi') \in \Pi(N')\}$ is a* pomset history preserving ST-branching τ-bisimulation *between N and N', denoted by $\mathcal{R} : N \underline{\leftrightarrow}^\tau_{pomhSTbr} N'$, if:*

1. $((\pi_N, \pi_N), (\pi_{N'}, \pi_{N'}), \emptyset) \in \mathcal{R}$.
2. $((\pi_E, \pi_P), (\pi'_E, \pi'_P), \beta) \in \mathcal{R} \ \Rightarrow\ \beta : vis(\rho_{C_E}) \simeq vis(\rho_{C'_E})$ *and* $\beta(vis(T_{C_P})) = vis(T_{C'_P})$.
3. $((\pi_E, \pi_P), (\pi'_E, \pi'_P), \beta) \in \mathcal{R},\ (\pi_E, \pi_P) \to (\tilde{\pi}_E, \tilde{\pi}_P)\ \Rightarrow$
   - *(a)* $((\tilde{\pi}_E, \tilde{\pi}_P), (\pi'_E, \pi'_P), \beta) \in \mathcal{R}$ *or*
   - *(b)* $\exists \tilde{\beta},\ (\bar{\pi}'_E, \bar{\pi}'_P),\ (\tilde{\pi}'_E, \tilde{\pi}'_P) :\ (\pi'_E, \pi'_P) \Rightarrow (\bar{\pi}'_E, \bar{\pi}'_P) \to (\tilde{\pi}'_E, \tilde{\pi}'_P),\ \tilde{\beta}|_{vis(T_{C_E})} = \beta,\ ((\pi_E, \pi_P), (\bar{\pi}'_E, \bar{\pi}'_P), \beta) \in \mathcal{R},\ ((\tilde{\pi}_E, \tilde{\pi}_P), (\tilde{\pi}'_E, \tilde{\pi}'_P), \tilde{\beta}) \in \mathcal{R}$.
4. *As item 3, but the roles of N and N' are reversed.*

Two nets N and N' are pomset history preserving ST-branching τ-bisimulation equivalent*, denoted by $N \underline{\leftrightarrow}^\tau_{pomhSTbr} N'$, if $\exists \mathcal{R} : N \underline{\leftrightarrow}^\tau_{pomhSTbr} N'$.*

Conflict preserving τ-equivalences

Let $\xi = \langle X, \prec, \#, l\rangle$ be a LES s.t. $l : X \to Act_\tau$. We denote $vis(X) = \{x \in X \mid l(x) \in Act\}$ and $vis(\xi) = \xi|_{vis(X)}$.

Definition 25 *A* visible MES-trace *of a net N, denoted by $vis(\xi)$, is an isomorphism class of LES $vis(\xi_O)$ for $\varpi = (O, \psi) \in \wp(N)$. We denote a set of* all visible MES-traces *of a net N by $VisMEStructs(N)$. Two nets N and N' are* MES-τ-conflict preserving equivalent*, denoted by $N \equiv^\tau_{mes} N'$, if $VisMEStructs(N) = VisMEStructs(N')$. Let us note that, due to uniqueness of maximal O-process, this is the same as to require $vis(\mathcal{E}(N)) = vis(\mathcal{E}(N'))$.*

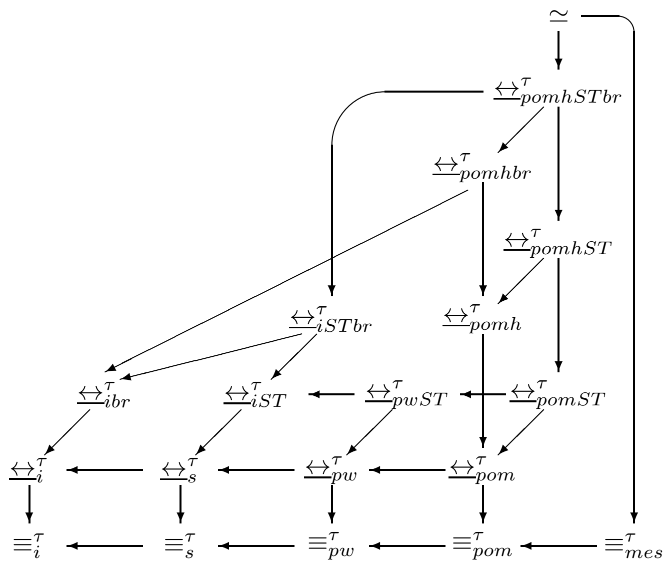

Figure 7: Interrelations of basic τ-equivalences

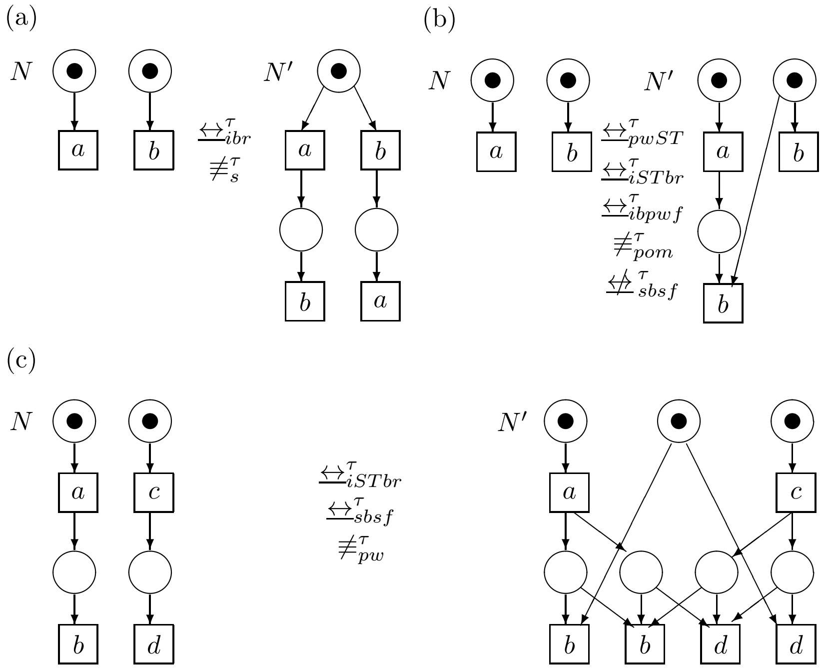

Figure 8: Examples of basic τ-equivalences

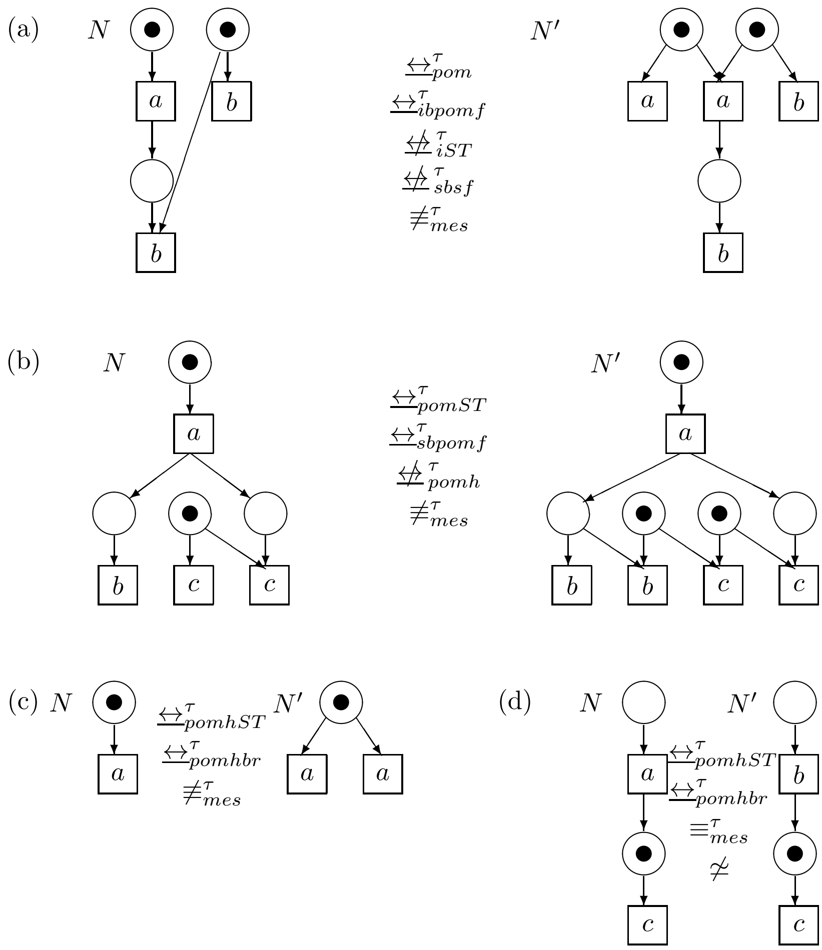

Figure 9: Examples of basic τ-equivalences (continued)

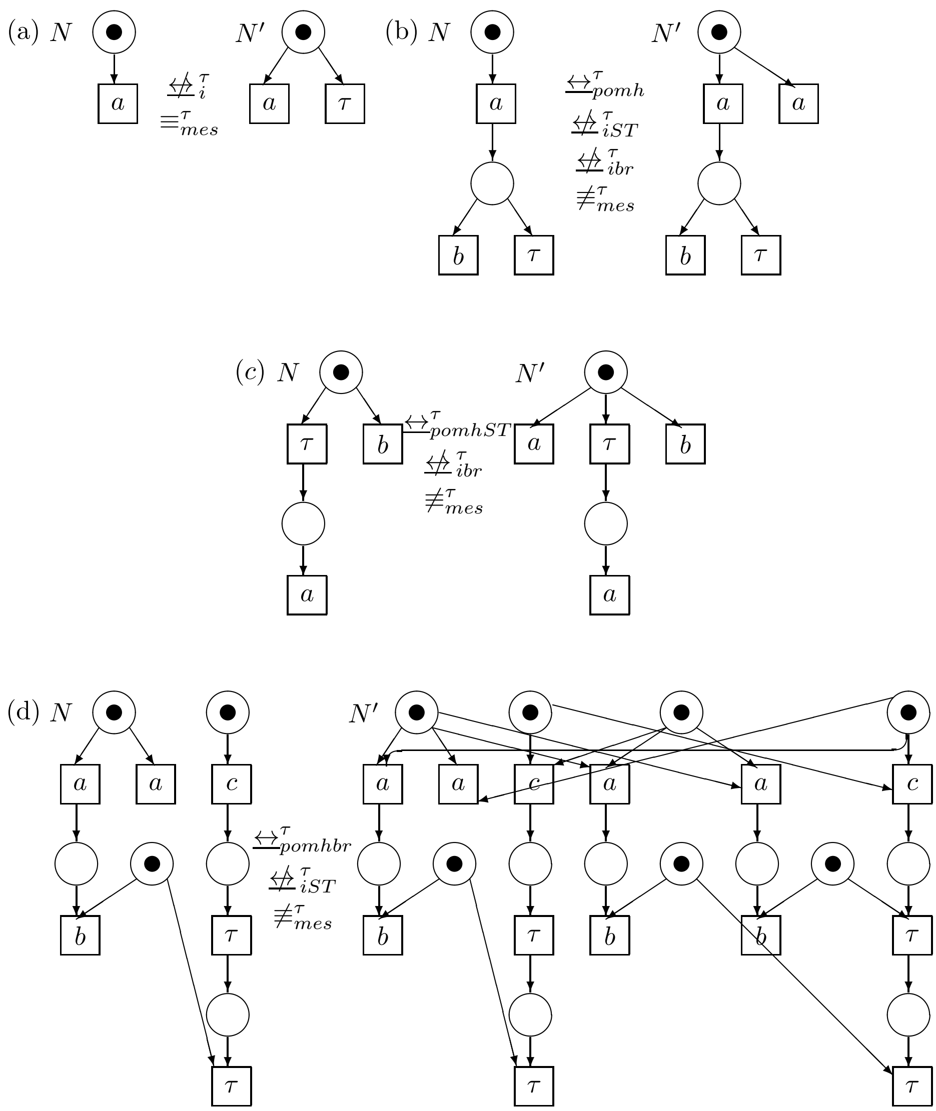

Figure 10: Examples of basic τ-equivalences (continued 2)

Sequential runs

Definition 26 *A* sequential run *of a net N is a pair (π, σ), where:*

- *a process $\pi \in \Pi(N)$ contains the information about causal dependencies of transitions which brought to this state;*
- *a sequence $\sigma \in T_C^*$ s.t. $\pi_N \xrightarrow{\sigma} \pi$, contains the information about the order in which the transitions occur which brought to this state.*

Let us denote the *set of all sequential runs* of a net N by $Runs(N)$.

The *initial* sequential run of a net N is a pair (π_N, ε), where ε is an empty sequence. Let us denote by $|\sigma|$ a *length* of a sequence σ.

Let $(\pi, \sigma),\ (\tilde{\pi}, \tilde{\sigma}) \in Runs(N)$. We write $(\pi, \sigma) \xrightarrow{\hat{\pi}} (\tilde{\pi}, \tilde{\sigma})$, if $\pi \xrightarrow{\hat{\pi}} \tilde{\pi},\ \exists \hat{\sigma} \in T_C^*\ \pi \xrightarrow{\hat{\sigma}} \tilde{\pi}$ and $\tilde{\sigma} = \sigma\hat{\sigma}$. We write $(\pi, \sigma) \to (\tilde{\pi}, \tilde{\sigma})$, if $(\pi, \sigma) \xrightarrow{\hat{\pi}} (\tilde{\pi}, \tilde{\sigma})$ for some $\hat{\pi}$.

Let $(\pi, \sigma) \in Runs(N),\ (\pi', \sigma') \in Runs(N')$ and $\sigma = v_1 \cdots v_n,\ \sigma' = v_1' \cdots v_n'$. Let us define a mapping $\beta_\sigma^{\sigma'} : T_C \to T_{C'}$ as follows: $\beta_\sigma^{\sigma'} = \{(v_i, v_i') \mid 1 \le i \le n\}$. Let $\beta_\varepsilon^\varepsilon = \emptyset$.

Let $(\pi, \sigma) \in Runs(N)$ and $\sigma = v_1 \cdots v_n,\ \pi_N \xrightarrow{v_1} \ldots \xrightarrow{v_i} \pi_i\ (1 \le i \le n)$.

Let us introduce the following notations:

- $\pi(0) = \pi_N$,
  $\pi(i) = \pi_i\ (1 \le i \le n)$;
- $\sigma(0) = \varepsilon$,
  $\sigma(i) = v_1 \cdots v_i\ (1 \le i \le n)$.

Back-forth τ-bisimulation equivalences

Definition 27 *Let N and N' be some nets. A relation $\mathcal{R} \subseteq Runs(N) \times Runs(N')$ is a* $\star$-back $\star\star$-forth τ-bisimulation *between N and N',* $\star, \star\star \in$ {interleaving, step, partial word, pomset}*, denoted by* $\mathcal{R} : N \underline{\leftrightarrow}^{\tau}_{\star b \star\star f} N'$, $\star, \star\star \in \{i, s, pw, pom\}$*, if:*

1. $((\pi_N, \varepsilon), (\pi_{N'}, \varepsilon)) \in \mathcal{R}$.
2. $((\pi, \sigma), (\pi', \sigma')) \in \mathcal{R}$
   - *(back)*
     $(\tilde{\pi}, \tilde{\sigma}) \xrightarrow{\hat{\pi}} (\pi, \sigma)$,
     (a) $|vis(T_{\widehat{C}})| = 1$*, if* $\star = i$*;*
     (b) $vis(\prec_{\widehat{C}}) = \emptyset$*, if* $\star = s$*;*
     $\Rightarrow\ \exists (\tilde{\pi}', \tilde{\sigma}') :\ (\tilde{\pi}', \tilde{\sigma}') \xrightarrow{\hat{\pi}'} (\pi', \sigma'),\ ((\tilde{\pi}, \tilde{\sigma}), (\tilde{\pi}', \tilde{\sigma}')) \in \mathcal{R}$ *and*
     (a) $vis(\rho_{\widehat{C}'}) \sqsubseteq vis(\rho_{\widehat{C}})$*, if* $\star = pw$*;*
     (b) $vis(\rho_{\widehat{C}}) \simeq vis(\rho_{\widehat{C}'})$*, if* $\star \in \{i, s, pom\}$*;*
   - *(forth)*
     $(\pi, \sigma) \xrightarrow{\hat{\pi}} (\tilde{\pi}, \tilde{\sigma})$,
     (a) $|vis(T_{\widehat{C}})| = 1$*, if* $\star\star = i$*;*
     (b) $vis(\prec_{\widehat{C}}) = \emptyset$*, if* $\star\star = s$*;*
     $\Rightarrow\ \exists (\tilde{\pi}', \tilde{\sigma}') :\ (\pi', \sigma') \xrightarrow{\hat{\pi}'} (\tilde{\pi}', \tilde{\sigma}'),\ ((\tilde{\pi}, \tilde{\sigma}), (\tilde{\pi}', \tilde{\sigma}')) \in \mathcal{R}$ *and*
     (a) $vis(\rho_{\widehat{C}'}) \sqsubseteq vis(\rho_{\widehat{C}})$*, if* $\star\star = pw$*;*
     (b) $vis(\rho_{\widehat{C}}) \simeq vis(\rho_{\widehat{C}'})$*, if* $\star\star \in \{i, s, pom\}$.
3. *As item 2, but the roles of N and N' are reversed.*

Two nets N and N' are $\star$-back $\star\star$-forth τ-bisimulation equivalent, $\star, \star\star \in$ {interleaving, step, partial word, pomset}*, denoted by* $N \underline{\leftrightarrow}^{\tau}_{\star b \star\star f} N'$*, if* $\exists \mathcal{R} : N \underline{\leftrightarrow}^{\tau}_{\star b \star\star f} N'$, $\star, \star\star \in \{i, s, pw, pom\}$.

$$\begin{tikzcd}
\underline{\leftrightarrow}^{\tau}_{pombif} \arrow[d] & \underline{\leftrightarrow}^{\tau}_{pombsf} \arrow[l] \arrow[d] & \underline{\leftrightarrow}^{\tau}_{pombpwf} \arrow[l] \arrow[d] & \underline{\leftrightarrow}^{\tau}_{pombpomf} \arrow[l] \arrow[d] \\
\underline{\leftrightarrow}^{\tau}_{pwbif} \arrow[d] & \underline{\leftrightarrow}^{\tau}_{pwbsf} \arrow[l] \arrow[d] & \underline{\leftrightarrow}^{\tau}_{pwbpwf} \arrow[l] \arrow[d] & \underline{\leftrightarrow}^{\tau}_{pwbpomf} \arrow[l] \arrow[d] \\
\underline{\leftrightarrow}^{\tau}_{sbif} \arrow[d] & \underline{\leftrightarrow}^{\tau}_{sbsf} \arrow[l] \arrow[d] & \underline{\leftrightarrow}^{\tau}_{sbpwf} \arrow[l] \arrow[d] & \underline{\leftrightarrow}^{\tau}_{sbpomf} \arrow[l] \arrow[d] \\
\underline{\leftrightarrow}^{\tau}_{ibif} & \underline{\leftrightarrow}^{\tau}_{ibsf} \arrow[l] & \underline{\leftrightarrow}^{\tau}_{ibpwf} \arrow[l] & \underline{\leftrightarrow}^{\tau}_{ibpomf} \arrow[l]
\end{tikzcd}$$

Figure 11: Merging of back-forth τ-bisimulation equivalences

$$\begin{tikzcd}
 & & & \underline{\leftrightarrow}^{\tau}_{pombpomf} \arrow[d] \\
 & \underline{\leftrightarrow}^{\tau}_{sbsf} \arrow[d] & \underline{\leftrightarrow}^{\tau}_{sbpwf} \arrow[l] \arrow[d] & \underline{\leftrightarrow}^{\tau}_{sbpomf} \arrow[l] \arrow[d] \\
\underline{\leftrightarrow}^{\tau}_{ibif} & \underline{\leftrightarrow}^{\tau}_{ibsf} \arrow[l] & \underline{\leftrightarrow}^{\tau}_{ibpwf} \arrow[l] & \underline{\leftrightarrow}^{\tau}_{ibpomf} \arrow[l]
\end{tikzcd}$$

Figure 12: Interrelations of back-forth τ-bisimulation equivalences

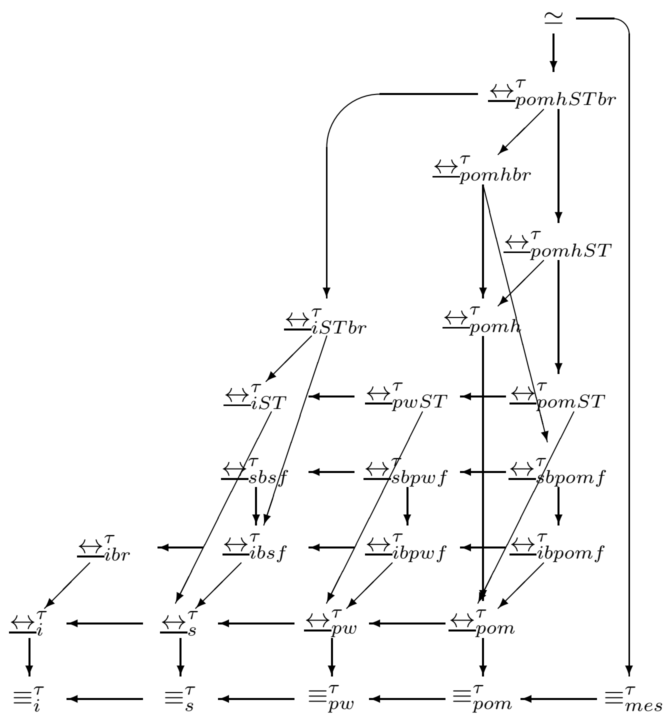

Figure 13: Interrelations of back-forth τ-bisimulation equivalences with basic τ-equivalences

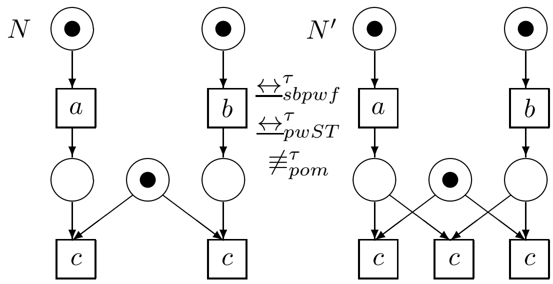

Figure 14: Example of back-forth τ-bisimulation equivalences

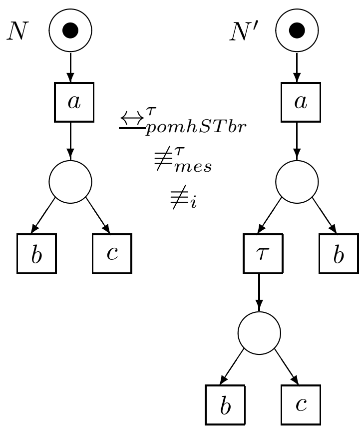

Figure 15: Example of interrelations of equivalences and τ-equivalences

SM-refinements

Definition 28 *An* SM-net *is a net* $D = \langle P_D, T_D, F_D, l_D, M_D \rangle$ *s.t.:*

1. $\forall t \in T_D\ |{}^\bullet t| = |t^\bullet| = 1$, *i.e. each transition has exactly one input and one output place;*
2. $\exists p_{in},\ p_{out} \in P_D$ *s.t.* $p_{in} \neq p_{out}$ *and* ${}^\circ D = \{p_{in}\},\ D^\circ = \{p_{out}\}$, *i.e. net* D *has unique input and unique output place.*
3. $M_D = \{p_{in}\}$, *i.e. at the beginning there is unique token in* p_{in}.

Definition 29 *Let* $N = \langle P_N, T_N, F_N, l_N, M_N \rangle$ *be some net,* $a \in l_N(T_N)$ *and* $D = \langle P_D, T_D, F_D, l_D, M_D \rangle$ *be SM-net. An* SM-refinement, *denoted by* $ref(N, a, D)$, *is (up to isomorphism) a net* $\overline{N} = \langle P_{\overline{N}}, T_{\overline{N}}, F_{\overline{N}}, l_{\overline{N}}, M_{\overline{N}} \rangle$, *where:*

- $P_{\overline{N}} = P_N \cup \{\langle p, u\rangle \mid p \in P_D \setminus \{p_{in}, p_{out}\},\ u \in l_N^{-1}(a)\}$;
- $T_{\overline{N}} = (T_N \setminus l_N^{-1}(a)) \cup \{\langle t, u\rangle \mid t \in T_D,\ u \in l_N^{-1}(a)\}$;
- $F_{\overline{N}}(\bar{x}, \bar{y}) = \begin{cases} F_N(\bar{x}, \bar{y}), & \bar{x}, \bar{y} \in P_N \cup (T_N \setminus l_N^{-1}(a)); \\ F_D(x, y), & \bar{x} = \langle x, u\rangle,\ \bar{y} = \langle y, u\rangle,\ u \in l_N^{-1}(a); \\ F_N(\bar{x}, u), & \bar{y} = \langle y, u\rangle,\ \bar{x} \in {}^\bullet u,\ u \in l_N^{-1}(a),\ y \in p_{in}^\bullet; \\ F_N(u, \bar{y}), & \bar{x} = \langle x, u\rangle,\ \bar{y} \in {}^\bullet u,\ u \in l_N^{-1}(a),\ x \in {}^\bullet p_{out}; \\ 0, & otherwise; \end{cases}$
- $l_{\overline{N}}(\bar{u}) = \begin{cases} l_N(\bar{u}), & \bar{u} \in T_N \setminus l_N^{-1}(a); \\ l_D(t), & \bar{u} = \langle t, u\rangle,\ t \in T_D,\ u \in l_N^{-1}(a); \end{cases}$
- $M_{\overline{N}}(p) = \begin{cases} M_N(p), & p \in P_N; \\ 0, & otherwise. \end{cases}$

An equivalence is *preserved by refinements*, if equivalent nets remain equivalent after applying any refinement operator to them accordingly.

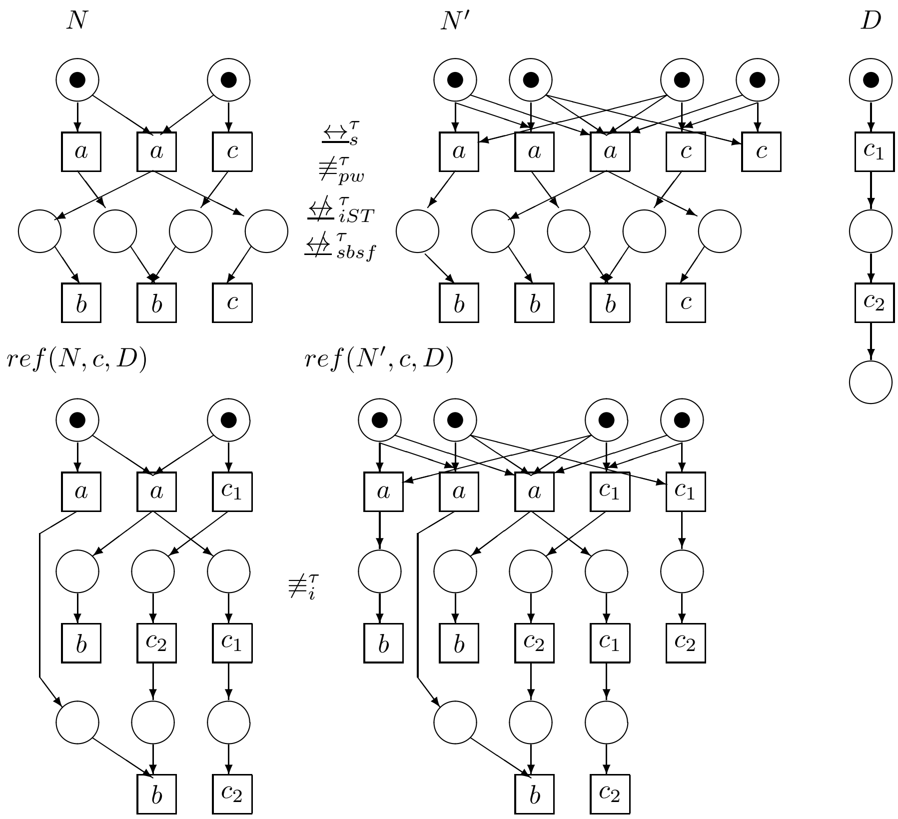

Figure 16: The τ-equivalences between $\equiv_i^\tau$ and $\underline{\leftrightarrow}_s^\tau$ are not preserved by SM-refinements

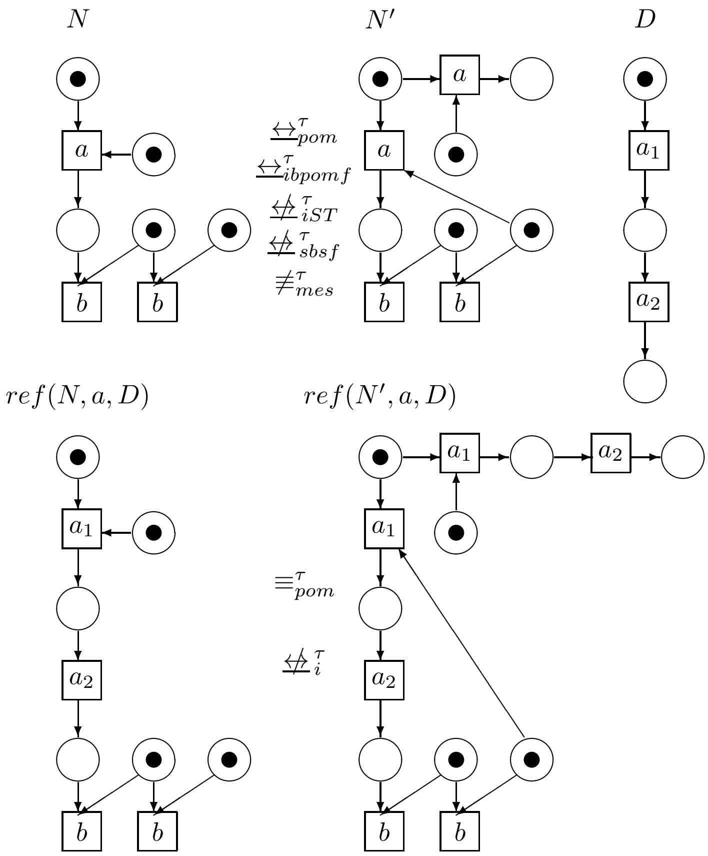

Figure 17: The τ-equivalences between $\underline{\leftrightarrow}^{\tau}_{i}$ and $\underline{\leftrightarrow}^{\tau}_{pom}$ are not preserved by SM-refinements

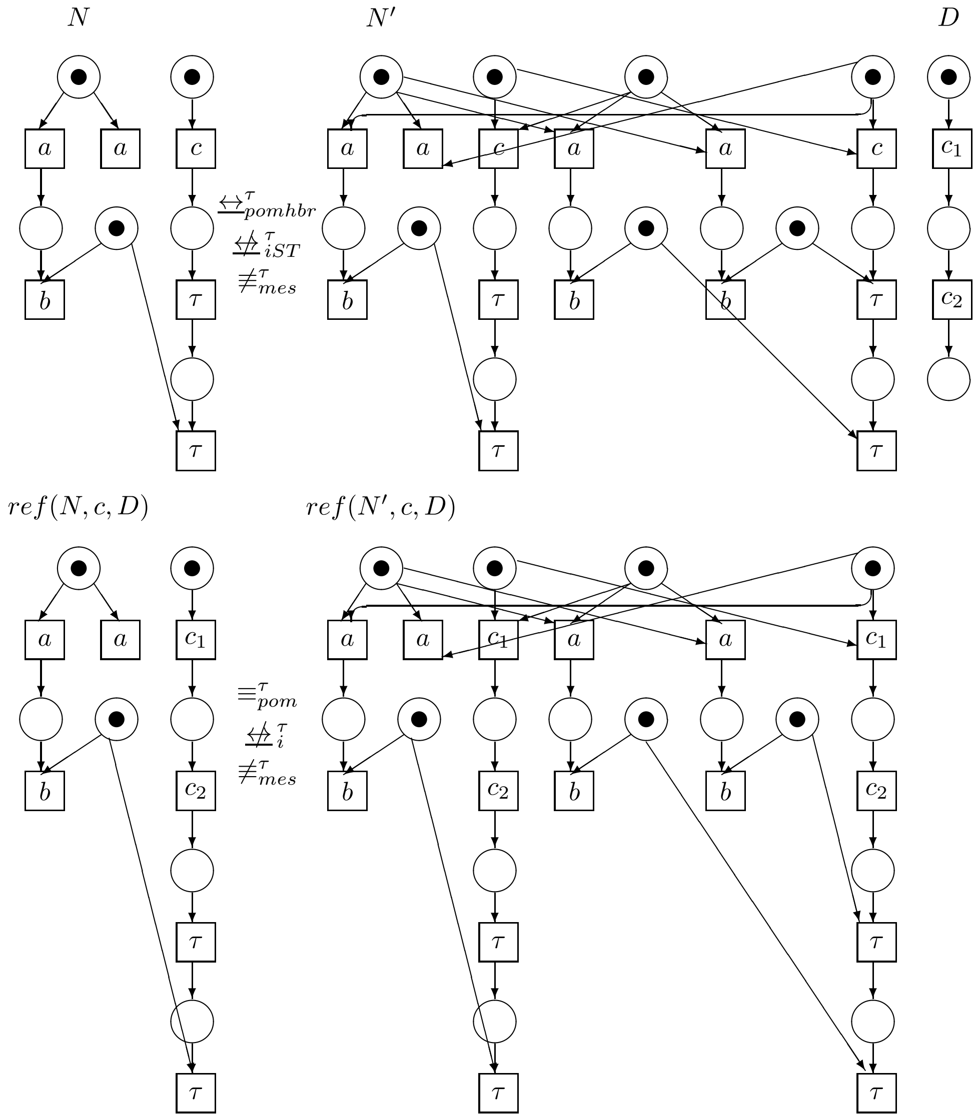

Figure 18: The τ-equivalences between $\underline{\leftrightarrow}^{\tau}_{i}$ and $\underline{\leftrightarrow}^{\tau}_{pomhbr}$ are not preserved by SM-refinements

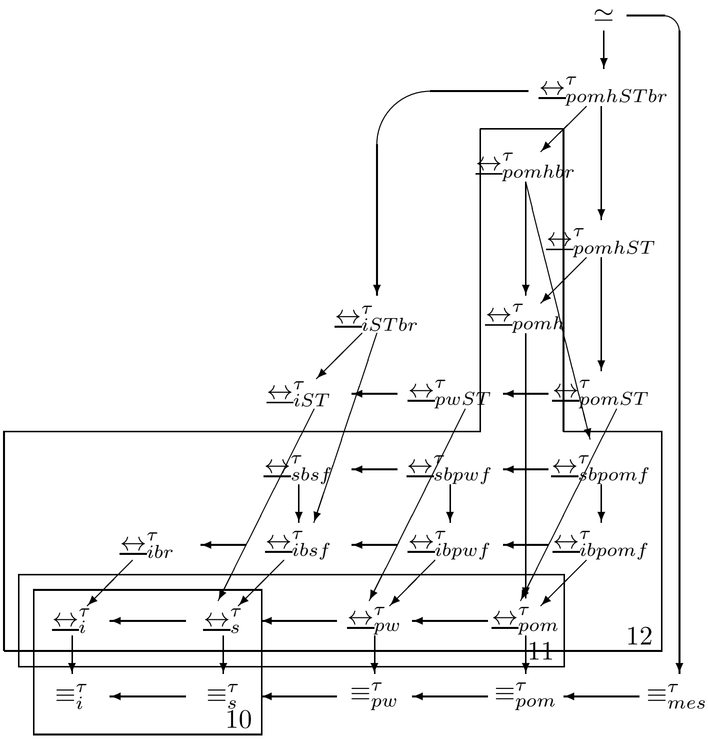

Figure 19: The τ-equivalences which are not preserved by SM-refinements

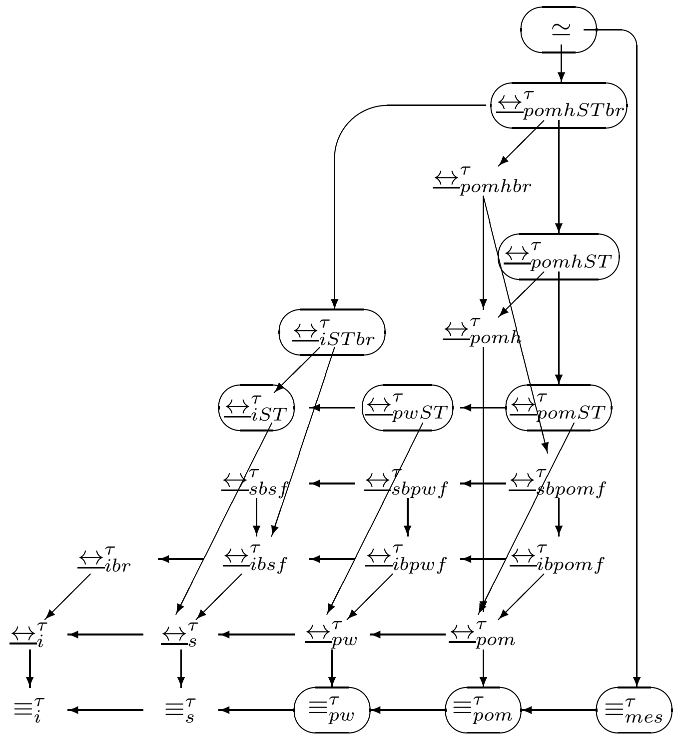

Figure 20: Preservation of the τ-equivalences by SM-refinements

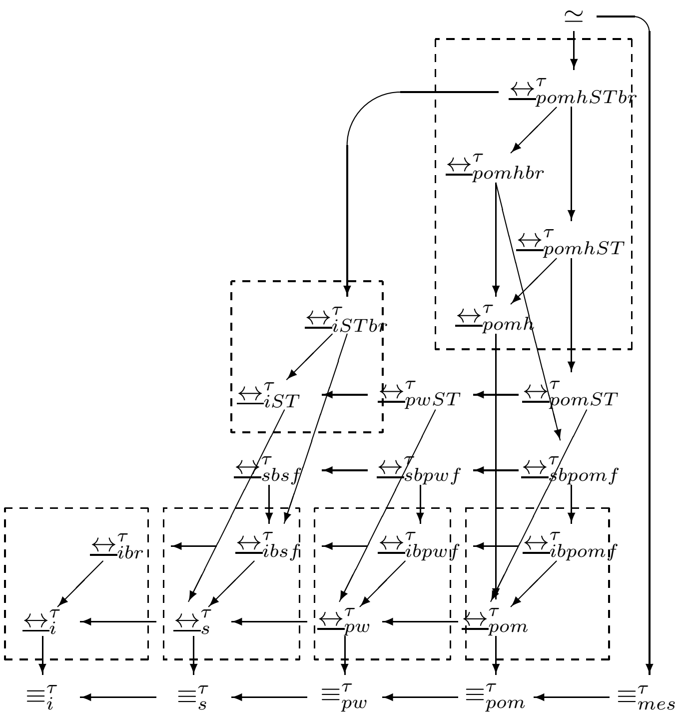

Figure 21: Merging of the τ-equivalences on nets without silent transitions

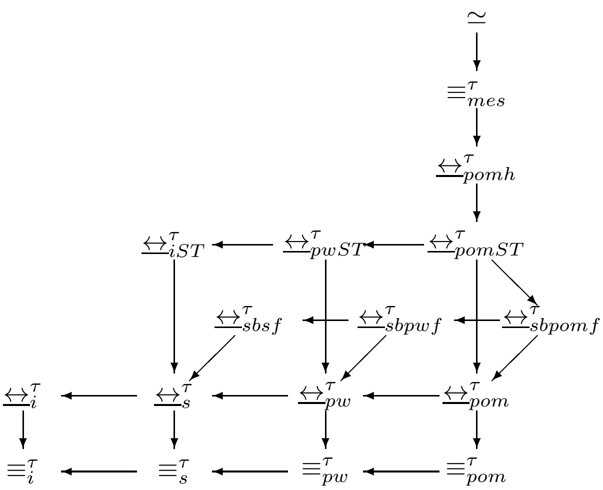

Figure 22: Interrelations of the τ-equivalences on nets without silent transitions

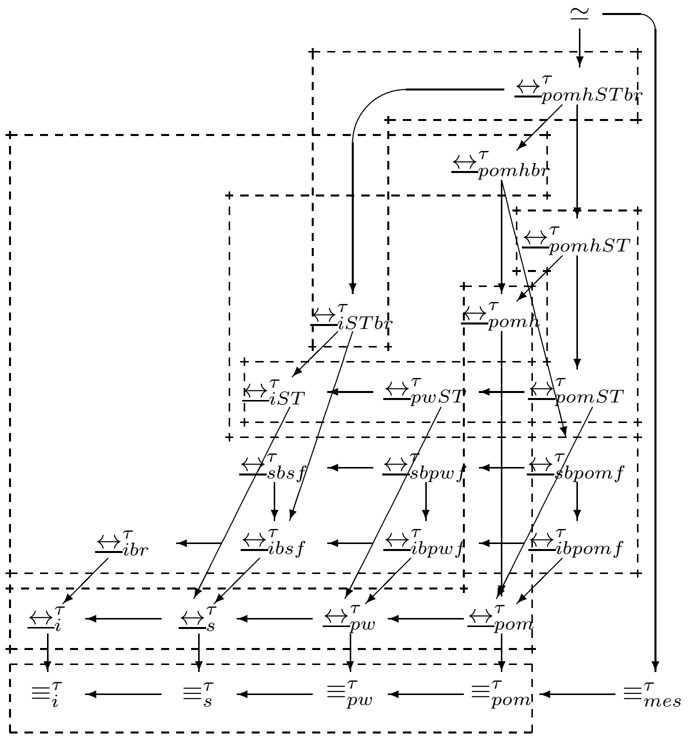

Figure 23: Merging of the τ-equivalences on sequential nets

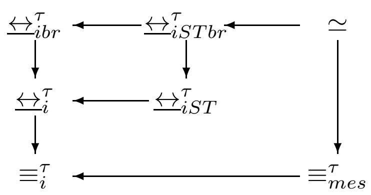

Figure 24: Interrelations of the τ-equivalences on sequential nets