Investigating Equivalence Notions for Time Petri Nets *

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An important ingredient of every theory of concurrency is a notion of equivalence between systems. Typically, equivalences are used in the setting of specification and verification both to compare two distinct systems and to reduce the structure of a system. Over the past several years, a variety of equivalences — most notably, perphars, trace and bisimulation ones — have been promoted, and the relationship between them has been quite well-understood (see, for example, [4]).

Those untimed equivalences abstract away from timed aspects of system behaviours. Recently, a growing interest can be observed in modelling real-time systems which imply a need of a representation of the lapse of time. Several formal methods for specifying and reasoning about real-time systems have been proposed in the last years, whereas the incorporation of real time into equivalence notions is less advanced. There are a few papers (see [1, 3] among others) where decidability questions of timed equivalences are investigated. In these studies, real-time systems are represented by timed automata, containing fictitious time measuring elements called clocks. However, concurrency can not be modelled directly by such timed states graphs. On the other hand, time Petri nets (time nets) were considered in [5] to model real-time systems over dense time domain. Our main point here is to introduce timed, untimed and region equivalences in the trace (denoted by '$\equiv$') and bisimulation (denoted by '$\underline{\leftrightarrow}$') cases and establish the interrelations between these notions in the framework of time nets. Timed equivalences [3] (subscribed by 't') can measure the exact real-numbered duration of every delay, whereas untimed ones [3] (subscribed by 'u') do not. Region equivalences (subscribed by 'r') partition the states of a time Petri net into the so-called 'regions' [1]. We also treat weak variants [4] of these equivalences (superscribed by 'τ') which take into consideration 'invisible' nature of the silent action τ.

The following theorem establishes the interrelations between the timed and untimed variants of the equivalences.

Theorem 1 *Let* $\leftrightarrow, \twoheadleftarrow\!\!\!\twoheadrightarrow \in \{\equiv_t, \underline{\leftrightarrow}_t, \equiv_u, \underline{\leftrightarrow}_u\}$ *and* $\star, \star\star \in \{., \tau\}$ *(the symbol '.' denotes "nothing"). For time nets* N *and* N'*:* $N \leftrightarrow^\star N' \;\Rightarrow\; N \twoheadleftarrow\!\!\!\twoheadrightarrow^{\star\star} N'$ *iff in the graph in Figure 1 there exists a directed path from* $\leftrightarrow^\star$ *to* $\twoheadleftarrow\!\!\!\twoheadrightarrow^{\star\star}$*.*

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$$
\begin{tikzcd}
& \underline{\leftrightarrow}_u \arrow[dl] \arrow[dd] & & \underline{\leftrightarrow}_t \arrow[ll] \arrow[dl] \arrow[dd] \\
\underline{\leftrightarrow}_u^\tau \arrow[dd] & & \underline{\leftrightarrow}_t^\tau \arrow[ll] \arrow[dd] & \\
& \equiv_u \arrow[dl] & & \equiv_t \arrow[ll] \arrow[dl] \\
\equiv_u^\tau & & \equiv_t^\tau \arrow[ll] &
\end{tikzcd}
$$

Figure 1: The interrelations of the equivalences

We next show the coincidence of the timed equivalences with the region ones. This provides a tool to reduce the number of states of a time net implying the simplification of timed equivalences checking.

Theorem 2 *Let* $\leftrightarrow \in \{\equiv, \underline{\leftrightarrow}\}$ *and* $\star \in \{., \tau\}$ *(the symbol '.' denotes "nothing"). For time nets* N *and* N'*:* $N \leftrightarrow_t^\star N' \;\Leftrightarrow\; N \leftrightarrow_r^\star N'$.

For untime nets, a subclass of time nets obtained by taking delay times of the transitions equal to zero's, the coincidence of the timed and untimed variants of the equivalences is established.

Theorem 3 *Let* $\leftrightarrow \in \{\equiv, \underline{\leftrightarrow}\}$ *and* $\star \in \{., \tau\}$ *(the symbol '.' denotes "nothing"). For untime nets* N *and* N'*:* $N \leftrightarrow_u^\star N' \;\Leftrightarrow\; N \leftrightarrow_t^\star N'$.

We finally treat the question of preservation of the considered equivalences under a special case of transition refinement (a modification of SM-refinement [2] for time nets, where transitions are replaced by time state-machine nets). The following theorem demonstrates which of the equivalences are preserved under time SM-refinement.

Theorem 4 *Let* $\leftrightarrow \in \{\underline{\leftrightarrow}_t, \underline{\leftrightarrow}_u\}$*. For time nets* N *and* N' *s.t. some their transitions are labelled by the visible action* a *and time SM-net* D*:* $N \leftrightarrow N' \;\Rightarrow\; ref(N, a, D) \leftrightarrow ref(N', a, D)$.

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