Labeled DTSPNs as a semantic area for stochastic process algebras

Igor V. Tarasyuk

A.P. Ershov Institute of Informatics Systems

Siberian Division of the Russian Academy of Sciences

6, Acad. Lavrentiev pr., Novosibirsk 630090, Russia

itar@iis.nsk.su

www.iis.nsk.su/persons/itar

Abstract: Labeled discrete time stochastic Petri nets (LDTSPNs) are proposed.

The visible behavior of LDTSPNs is described by transition labels. The dynamic behavior is defined.

Trace and bisimulation probabilistic equivalences are considered. A diagram of their interrelations is presented.

Stochastic algebra of finite processes $StAFP_0$ is proposed with a net semantics based on a subclass of LDTSPNs.

Keywords: stochastic Petri nets, step semantics, probabilistic equivalences, bisimulation, stochastic process algebras.

Contents

Previous work

Transition labeling

- CTSPNs [Buc95]
- GSPNs [Buc98]
- DTSPNs [BT00]

Equivalences

- Stochastic automata (SAs) [Buc99]
- Probabilistic transition systems (PTSs) [BM89,Chr90,LS91,BH97,KN98]
- CTMCs [HR94,Hil94]
- CTSPNs [Buc95]
- GSPNs [Buc98]
- Markov process algebras (MPAs) [Buc94]
- Stochastic event structures (SESs) [MCW03]

Process algebras

- AFP_0 [KCh85]
- PBC [BDH92]

Formal model

Definition 1 *A Labeled discrete time stochastic Petri net (LDTSPN) is a tuple* $N = (P_N, T_N, W_N, \Omega_N, L_N, M_N)$*:*

- P_N *and* T_N *are finite sets of places and transitions* ($P_N \cup T_N \neq \emptyset,\ P_N \cap T_N = \emptyset$)*;*
- $W_N : (P_N \times T_N) \cup (T_N \times P_N) \to I\!N$ *is the arc weight function;*
- $\Omega_N : T_N \to (0; 1]$ *is the transition conditional probability function;*
- $L_N : T_N \to Act_\tau$ *is the transition labeling function* ($Act_\tau = Act \cup \{\tau\}$)*;*
- $M_N \in I\!N_f^{P_N}$ *is the initial marking.*

Let M be a marking of a LDTSPN $N = (P_N, T_N, W_N, \Omega_N, L_N, M_N)$. Then $t \in Ena(M)$ fires in the next time moment with probability $\Omega_N(t)$, if no other transition is enabled in M: conditional probability.

Conditional probability to fire in a marking M for a transition set (not a multiset) $U \subseteq Ena(M)$ s.t. ${}^\bullet U \subseteq M$:

$$PF(U, M) = \prod_{t \in U} \Omega_N(t) \cdot \prod_{t \in Ena(M) \setminus U} (1 - \Omega_N(t)).$$

Concurrent transition firings at discrete time moments.

LDTSPNs have *step* semantics.

Behavior of the model

Let $M \in \mathbb{N}_f^{P_N}$ be a marking of a LDTSPN N and $U \subseteq Ena(M)$ be a set of transitions s.t. ${}^\bullet U \subseteq M$.

Firing of U changes marking M by $\widetilde{M} = M - {}^\bullet U + U^\bullet$, $M \xrightarrow{U}_{\mathcal{P}} \widetilde{M}$.

The probability $\mathcal{P} = PT(U, M)$ is

$$PT(U, M) = \frac{PF(U, M)}{\sum_{\{V \subseteq Ena(M) \mid {}^\bullet V \subseteq M\}} PF(V, M)}.$$

We write $M \xrightarrow{U} \widetilde{M}$ if $\exists \mathcal{P} > 0\ M \xrightarrow{U}_{\mathcal{P}} \widetilde{M}$.

For $A \in \mathbb{N}_f^{Act_\tau}$ we define $vis(A) = \sum_{a \in A \cap Act} a$.

Let $A \in \mathbb{N}_f^{Act}$. $M \xrightarrow{A}_{\mathcal{P}} \widetilde{M}$ is a step starting in M, performing transitions that are *visibly* labeled by A and ending in $\widetilde{M}$.

The probability $\mathcal{P} = PS(A, M, \widetilde{M})$ is

$$PS(A, M, \widetilde{M}) = \sum_{\{U \subseteq Ena(M) \mid M \xrightarrow{U} \widetilde{M},\ vis(L_N(U)) = A\}} PT(U, M).$$

We write $M \xrightarrow{A} \widetilde{M}$ if $\exists \mathcal{P} > 0\ M \xrightarrow{A}_{\mathcal{P}} \widetilde{M}$.

Definition 2 *For a LDTSPN N we define the following notions.*

- *The* reachability set $RS(N)$ *is the minimal set of markings s.t.*
  - $M_N \in RS(N)$;
  - *if* $M \in RS(N)$ *and* $M \stackrel{A}{\to} \widetilde{M}$ *then* $\widetilde{M} \in RS(N)$.
- *The* reachability graph $RG(N)$ *is a directed labeled graph with*
  - *the set of nodes* $RS(N)$;
  - *an arc labeled by* $A, \mathcal{P}$ *between nodes* M *and* $\widetilde{M}$ *if* $M \stackrel{A}{\to}_{\mathcal{P}} \widetilde{M}$ *and* $\mathcal{P} > 0$.
- *The* underlying Discrete Time Markov Chain (DTMC) $DTMC(N)$ *is a DTMC with*
  - *the state space* $RS(N)$;
  - *a transition* $M \to_{\mathcal{P}} \widetilde{M}$ *if at least one arc between* M *and* $\widetilde{M}$ *exists in* $RG(N)$. *The probability* $\mathcal{P} = PM(M, \widetilde{M})$ *is*

$$PM(M, \widetilde{M}) = \sum_{A \in I\!N_f^{Act}} PS(A, M, \widetilde{M}).$$

An *internal step* $M \stackrel{\emptyset}{\rightarrow}_{\mathcal{P}} \widetilde{M}$ with $\mathcal{P} > 0$ takes place when

- $\widetilde{M}$ is reachable from M by firing a set of internal transitions or
- no transition fires.

The *probability of reaching* $\widetilde{M}$ *from* M *by* k *internal steps* is

$$PS^k(\emptyset, M, \widetilde{M}) = \begin{cases} \sum_{\overline{M} \in RS(N)} PS^{k-1}(\emptyset, M, \overline{M}) \cdot PS(\emptyset, \overline{M}, \widetilde{M}) & \text{if } k \geq 1; \\ 1 & \text{if } k = 0 \text{ and } M = \widetilde{M}; \\ 0 & \text{otherwise.} \end{cases}$$

The *probability of reaching* $\widetilde{M}$ *from* M *by internal steps* is

$$PS^*(\emptyset, M, \widetilde{M}) = \sum_{k=0}^{\infty} PS^k(\emptyset, M, \widetilde{M}).$$

The *probability of reaching* $\widetilde{M}$ *from* M *by internal steps, followed by an visible step* A is

$$PS^*(A, M, \widetilde{M}) = \sum_{\overline{M} \in RS(N)} PS^*(\emptyset, M, \overline{M}) \cdot PS(A, \overline{M}, \widetilde{M}).$$

New transition relation: $M \stackrel{A}{\twoheadrightarrow}_{\mathcal{P}} \widetilde{M}$ where $\mathcal{P} = PS^*(A, M, \widetilde{M})$ and $A \neq \emptyset$.

We write $M \stackrel{A}{\twoheadrightarrow} \widetilde{M}$ if $\exists \mathcal{P} > 0\ M \stackrel{A}{\twoheadrightarrow}_{\mathcal{P}} \widetilde{M}$.

For $A = \{a\}$ we write $M \stackrel{a}{\twoheadrightarrow}_{\mathcal{P}} \widetilde{M}$ and $M \stackrel{a}{\twoheadrightarrow} \widetilde{M}$.

$RS^*(N)$ and $RG^*(N)$ are the *visible reachability set* and *graph*.

The *visible underlying DTMC* $DTMC^*(N)$ with state space $RS^*(N)$ and transition probabilities

$$PM^*(M, \widetilde{M}) = \sum_{A \in \mathbb{N}_f^{Act} \setminus \emptyset} PS^*(A, M, \widetilde{M}).$$

We write $M \twoheadrightarrow_{\mathcal{P}} \widetilde{M}$ if $\mathcal{P} = PM^*(M, \widetilde{M})$.

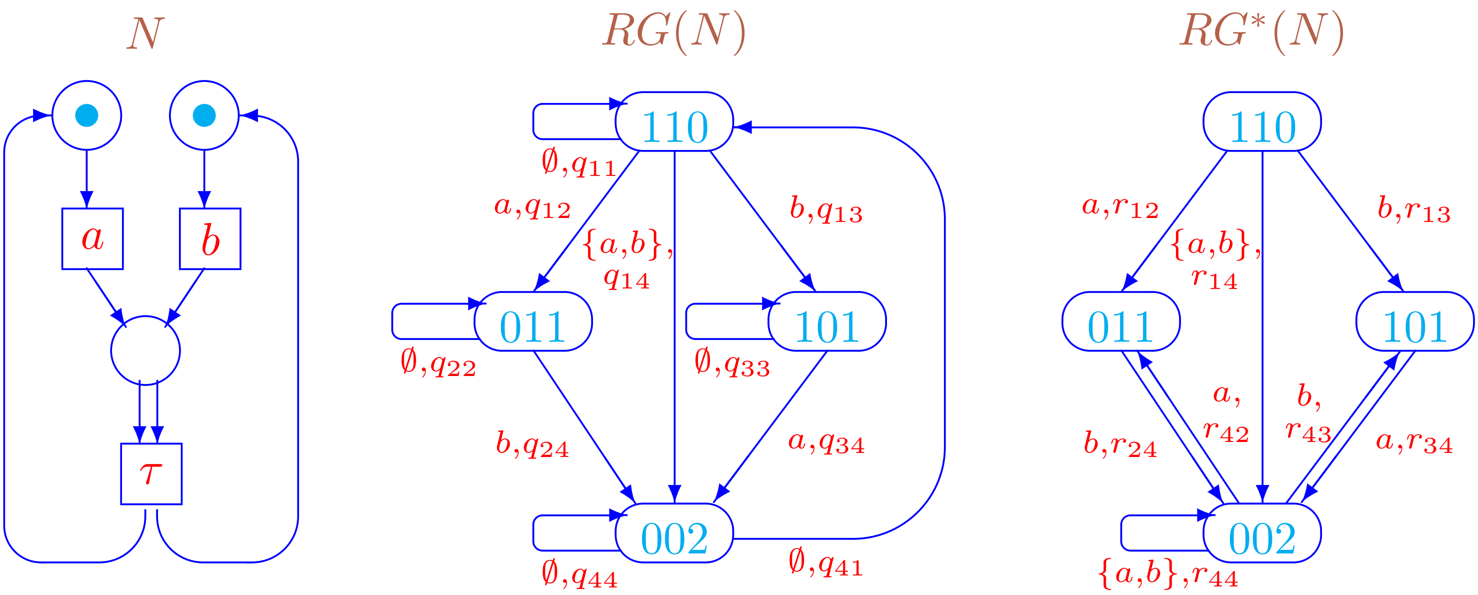
A *trap* is a loop of internal transitions starting and ending in some marking M which occurs with probability 1.

$PS^*(\emptyset, M, \widetilde{M})$ is finite as long as no traps exist.

If $PS^*(\emptyset, M, \widetilde{M})$ is finite, then $PS^*(A, M, \widetilde{M})$ defines a probability distribution:

$$\sum_{A \in \mathbb{N}_f^{Act} \setminus \emptyset} \sum_{\widetilde{M} \in RS^*(N)} PS^*(A, M, \widetilde{M}) = 1.$$

Example of LDTSPNs



A LDTSPN and the corresponding reachability graphs

$q_{11} = \overline{\Omega}_N(t_1) \cdot \overline{\Omega}_N(t_2)$ $\quad q_{12} = \Omega_N(t_1) \cdot \overline{\Omega}_N(t_2)$ $\quad q_{13} = \overline{\Omega}_N(t_1) \cdot \Omega_N(t_2)$

$q_{14} = \Omega_N(t_1) \cdot \Omega_N(t_2)$ $\quad q_{22} = \overline{\Omega}_N(t_2)$ $\quad q_{24} = \Omega_N(t_2)$

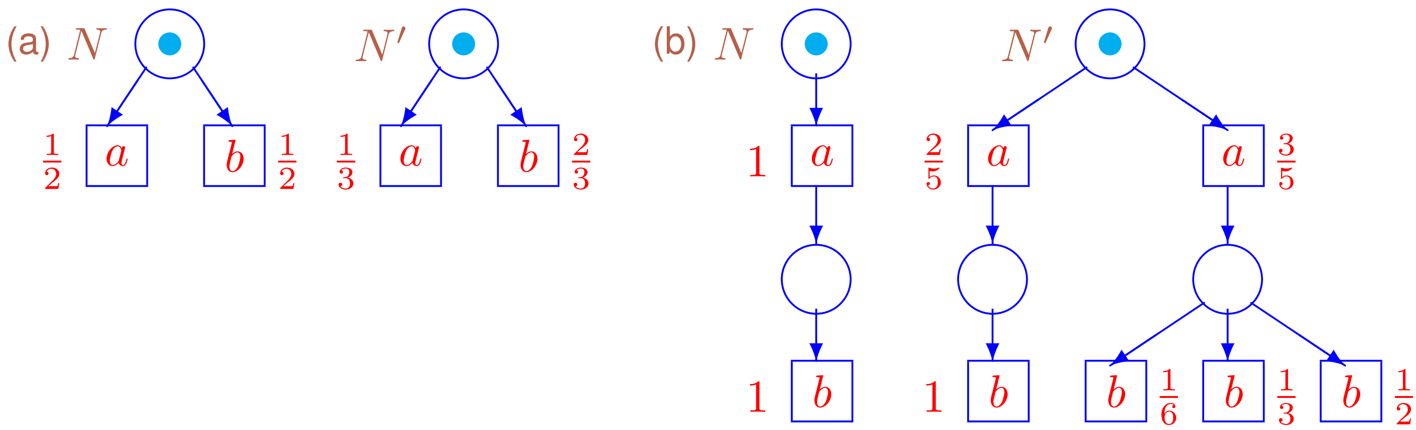
$q_{33} = \overline{\Omega}_N(t_1)$ $\quad q_{34} = \Omega_N(t_1)$ $\quad q_{41} = \Omega_N(t_3)$

$q_{44} = \overline{\Omega}_N(t_3)$

$r_{12} = r_{42} = \frac{q_{12}}{1-q_{11}}$ $\quad r_{13} = r_{43} = \frac{q_{13}}{1-q_{11}}$ $\quad r_{14} = r_{44} = \frac{q_{14}}{1-q_{11}}$

$r_{24} = 1$ $\quad r_{34} = 1$

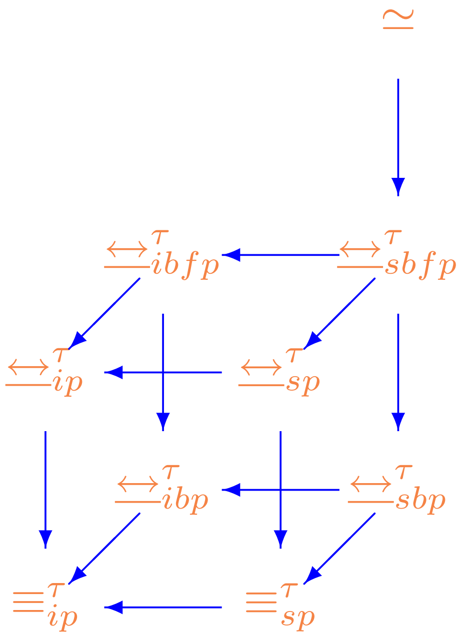
Properties of probabilistic relations



PP: Properties of probabilistic equivalences

- In Figure PP(a) LDTSPNs N and N' could not be related by any (even trace) probabilistic equivalence, since only in N' action a has probability $\frac{1}{3}$.
- In Figure PP(b) LDTSPNs N and N' are related by any (even bisimulation) probabilistic equivalence, since in our model probabilities of consequent actions are multiplied, and that of alternative ones are summarized.

Comparing the probabilistic τ-equivalences



Interrelations of the probabilistic τ-equivalences

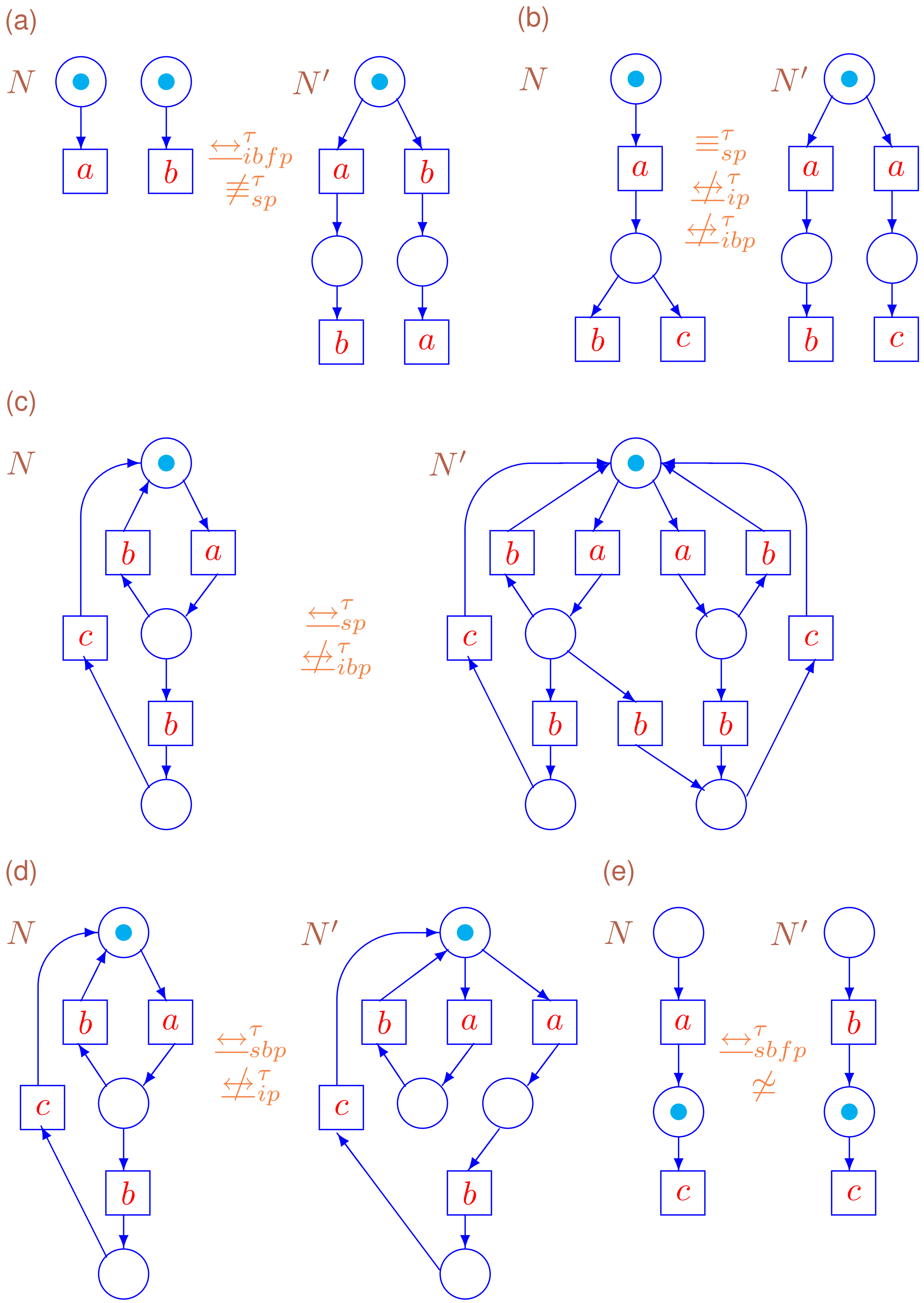
Proposition 1 *Let* $\star \in \{i, s\}$. *For LDTSPNs* N *and* N'

1. $N \underline{\leftrightarrow}^{\tau}_{\star p} N' \ \Rightarrow \ N \equiv^{\tau}_{\star p} N'$;
2. $N \underline{\leftrightarrow}^{\tau}_{\star bp} N' \ \Rightarrow \ N \equiv^{\tau}_{\star p} N'$;
3. $N \underline{\leftrightarrow}^{\tau}_{\star bfp} N' \ \Rightarrow \ N \underline{\leftrightarrow}^{\tau}_{\star p} N'$ *and* $N \underline{\leftrightarrow}^{\tau}_{\star bp} N'$.

Theorem 1 *Let* $\leftrightarrow, \twoheadleftarrow\!\!\twoheadrightarrow \in \{\equiv^{\tau}, \underline{\leftrightarrow}^{\tau}, \simeq\}$ *and* $\star, \star\star \in \{\_, ip, sp, ibp, sbp, ibfp, sbfp\}$. *For LDTSPNs* N *and* N'

$$N \leftrightarrow_{\star} N' \ \Rightarrow \ N \twoheadleftarrow\!\!\twoheadrightarrow_{\star\star} N'$$

iff in the graph in figure above there exists a directed path from $\leftrightarrow_{\star}$ *to* $\twoheadleftarrow\!\!\twoheadrightarrow_{\star\star}$.



S: Examples of the probabilistic τ-equivalences

- In Figure S(a), $N \underline{\leftrightarrow}^{\tau}_{ibfp} N'$, but $N \not\equiv^{\tau}_{sp} N'$, since only in the LDTSPN N' actions a and b cannot occur concurrently.
- In Figure S(b), $N \equiv^{\tau}_{sp} N'$, but $N \underline{\not\leftrightarrow}^{\tau}_{ip} N'$ and $N \underline{\not\leftrightarrow}^{\tau}_{ibp} N'$, since only in the LDTSPN N' an action a can occur so that no action b can occur afterwards.
- In Figure S(c), $N \underline{\leftrightarrow}^{\tau}_{sp} N'$, but $N \underline{\not\leftrightarrow}^{\tau}_{ibp} N'$, since only in N' there is a place with two input transitions labeled by b. Hence, the probability for a token to go to this place is always more than for that with only one input b-labeled transition.
- In Figure S(d), $N \underline{\leftrightarrow}^{\tau}_{sbp} N'$, but $N \underline{\not\leftrightarrow}^{\tau}_{ip} N'$, since only in the LDTSPN N' an action a can occur so that a sequence of actions bc cannot occur just after it.
- In Figure S(e), $N \underline{\leftrightarrow}^{\tau}_{sbfp} N'$ but $N \not\simeq N'$, since upper transitions of LDTSPNs N and N' are labeled by different actions (a and b).

Stochastic process algebra $StAFP_0$

Algebra of finite nondeterministic parallel processes AFP_0 [KCh85].

Specification of acyclic nets (A-nets, ANs).

Stochastic algebra of finite processes $StAFP_0$.

Specification of stochastic A-nets (SANs).

Syntax

An *activity* (a, ω):

- $a \in \mathit{Act}$ is the *action* label;
- $\omega \in (0; 1]$ is the *probability* of action a.

AP is the set of *all activities*.

Operations: *concurrency* $\|$, *precedence* $;$, *alternative* $\bigtriangledown$.

Definition 3 *Let* $(a, \omega) \in AP$. *A* formula *of* $StAFP_0$*:*

$$P ::= (a, \omega) \mid P \| P \mid P;P \mid P \bigtriangledown P.$$

$\mathbf{StAFP_0}$ is the set of *all formulas* of $StAFP_0$.

Semantics

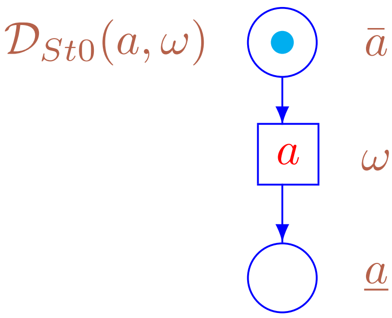
Formulas of $StAFP_0$ specify a subclass of LDTSPNs, *Stochastic A-nets (SANs)*: $T_N \subseteq Act,\ L_N = id_{T_N},\ M_N = {}^\bullet N$.

Thus, a SAN is specified by a quadruple $N = (P_N, T_N, W_N, \Omega_N)$.

The *net representation* of formulas, a mapping $\mathcal{D}_{St0}$ from $\mathbf{StAFP_0}$ to SANs.

Let $(a, \omega) \in AP$. An *atomic net* $\mathcal{D}_{St0}(a, \omega) = (P_N, T_N, W_N, \Omega_N)$, where

- $P_N = \{\bar{a}, \underline{a}\}$;
- $T_N = \{a\}$;
- $W_N = \{(\bar{a}, a), (a, \underline{a})\}$;
- $\Omega_N = \{(a, \omega)\}$.



An atomic net

Let $N = (P_N, T_N, W_N, \Omega_N)$ be a SAN and $Q, R \subseteq P_N$.

A *forming* operation $\otimes$:

$$Q \otimes R = \{q \cup r \mid q \in Q,\ r \in R\}.$$

The *merging* operation μ over a SAN $N = (P_N, T_N, W_N, \Omega_N)$ merges two sets of its places $Q, R \subseteq P$:

$$\mu(N, Q, R) = (\widetilde{P}_N, T_N, \widetilde{W}_N, \Omega_N), \text{ where}$$

- $\widetilde{P}_N = P_N \setminus (Q \cup R) \cup (Q \otimes R)$;
- $\forall t \in T_N\ \widetilde{W}_N(p, t) =$
$$\begin{cases} W_N(p,t), & p \in \widetilde{P}_N \setminus (Q \otimes R); \\ \max\{W_N(r,t), W_N(q,t)\}, & p = (q \cup r) \in Q \otimes R, \\ & q \in Q,\ r \in R. \end{cases}$$
$\forall t \in T_N\ \widetilde{W}_N(t, p) =$
$$\begin{cases} W_N(t,p), & p \in \widetilde{P}_N \setminus (Q \otimes R); \\ \max\{W_N(t,r), W_N(t,q)\}, & p = (q \cup r) \in Q \otimes R, \\ & q \in Q,\ r \in R. \end{cases}$$

Let $N = (P_N, T_N, W_N, \Omega_N)$ and $N' = (P_{N'}, T_{N'}, W_{N'}, \Omega_{N'})$ be two SANs. Net operations:

Concurrency $N \| N' = (P_N \cup P_{N'}, T_N \cup T_{N'}, W_N \cup W_{N'}, \Omega)$, where

$$\Omega(a) = \begin{cases} \Omega_N(a), & a \in T_N \setminus T_{N'}; \\ \Omega_{N'}(a), & a \in T_{N'} \setminus T_N; \\ \Omega_N(a) \cdot \Omega_{N'}(a), & a \in T_N \cap T_{N'}. \end{cases}$$

Precedence $N;N' = \mu(N \| N', N^{\bullet}, {}^{\bullet}N')$.

Alternative $N \bigtriangledown N' = \mu(\mu(N \| N', {}^{\bullet}N, {}^{\bullet}N'), N^{\bullet}, N'^{\bullet})$.

Nets N and N' combined by $;$ and $\bigtriangledown$ contain no equally named transitions.

Formulas P and P' combined by $;$ and $\bigtriangledown$ contain no identical actions.

Let $P, Q \in \mathbf{StAFP_0}$. The net representation of combined formulas:

1. $\mathcal{D}_{St0}(P \| Q) = \mathcal{D}_{St0}(P) \| \mathcal{D}_{St0}(Q)$;
2. $\mathcal{D}_{St0}(P;Q) = \mathcal{D}_{St0}(P);\mathcal{D}_{St0}(Q)$;
3. $\mathcal{D}_{St0}(P \triangledown Q) = \mathcal{D}_{St0}(P) \triangledown \mathcal{D}_{St0}(Q)$.

Definition 4 *Formulas P and P' are* semantic equivalent *in $StAFP_0$, $P =_{St0} P'$, if $\mathcal{D}_{St0}(P) \simeq \mathcal{D}_{St0}(P')$.*

Axiomatization

Let $P \in \mathbf{StAFP_0}$. The *structure* of P, $\phi_P \in \mathbf{AFP_0}$, specifies the non-stochastic process: replace each activity (a, ω) of P by a.

The *action probability function* Ω_P from actions contained in activities of P to $(0; 1]$. Let $(a, \omega_1), \ldots, (a, \omega_n)$ be *all* activities of P with action a. Then $\Omega_P(a) = \omega_1 \cdots \omega_n$.

The axiom system Θ_{St0}: in accordance with $=_{St0}$. Here $a \in Act$ and $P, Q, G \in \mathbf{StAFP_0}$.

1. Associativity

1.1 $P \| (Q \| R) = (P \| Q) \| R$

1.2 $P; (Q; R) = (P; Q); R$

1.3 $P \bigtriangledown (Q \bigtriangledown R) = (P \bigtriangledown Q) \bigtriangledown R$

2. Commutativity

2.1 $P \| Q = Q \| P$

2.2 $P \bigtriangledown Q = Q \bigtriangledown P$

3. Distributivity

3.1 $P; (Q \| R) = (P_1; Q) \| (P_2; R),\ \phi_P = \phi_{P_1} = \phi_{P_2},\ \Omega_P = \Omega_{P_1} \cdot \Omega_{P_2}$

3.2 $(P \| Q); R = (P; R_1) \| (Q; R_2),\ \phi_R = \phi_{R_1} = \phi_{R_2},\ \Omega_R = \Omega_{R_1} \cdot \Omega_{R_2}$

3.3 $P \bigtriangledown (Q \| R) = (P_1 \bigtriangledown Q) \| (P_2 \bigtriangledown R),\ \phi_P = \phi_{P_1} = \phi_{P_2},\ \Omega_P = \Omega_{P_1} \cdot \Omega_{P_2}$

4. Probability

4.1 $P = P_1 \| P_2,\ \phi_P = \phi_{P_1} = \phi_{P_2},\ \Omega_P = \Omega_{P_1} \cdot \Omega_{P_2}$

The axiom system Θ_{St0} is sound w.r.t. the equivalence $=_{St0}$.

A formula $P \in \mathbf{StAFP_0}$ is a *totally stratified* one iff $P = P_1 \| \cdots \| P_n,\ n \geq 1$ and each $P_i\ (1 \leq i \leq n)$ is a *primitive formula* i.e., does not contain $\|$.

Theorem 2 *Any formula $P \in \mathbf{StAFP_0}$ can be transformed (with the use of Θ_{St0}) into an equivalent (via $=_{St0}$) totally stratified one.*

The results obtained

- A new class of stochastic Petri nets with labeled transitions and a step semantics for transition firing (LDTSPNs).
- Equivalences for LDTSPNs which preserve interesting aspects of behavior and thus can be used

  to compare systems and to compute for a given one a minimal equivalent representation [Buc95].
- A diagram of interrelations for the equivalences.
- Stochastic algebra of finite processes $StAFP_0$ for specification of stochastic A-nets (SANs).
- A sound axiomatization of the net equivalence.

Further research

- Other equivalences in interleaving and step semantics:

  interleaving branching bisimulation [PRS92]

  (respecting conflicts with invisible transitions),

  back-forth bisimulations [NMV90,Pin93]

  (moving backward along history of computation).

- True concurrent equivalences:

  partial word and *pomset bisimulations* [PRS92,Vog92,MCW03]

  (partial order models of computation).

- More flexible process algebras:

  Petri box calculus (PBC) [BDH92]

  (infinite processes: recursion and iteration).

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