Performance analysis of the dining philosophers system in $dtsPBC$

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Abstract: Algebra $dtsPBC$ is a discrete time stochastic extension of finite Petri box calculus (PBC) enriched with iteration.

In this work, within dtsPBC, a method of modeling and performance evaluation based on stationary behaviour analysis for concurrent systems is outlined applied to the dining philosophers system.

Keywords: stochastic process algebra, Petri box calculus, discrete time, iteration, stationary behaviour, performance evaluation, dining philosophers system.

Contents

Introduction

Algebra PBC and its extensions

- *Petri box calculus* PBC [BDH92]
- *Time Petri box calculus* $tPBC$ [Kou00]
- *Timed Petri box calculus* $TPBC$ [MF00]
- *Stochastic Petri box calculus* $sPBC$ [MVF01,MVCC03]
- *Ambient Petri box calculus* $APBC$ [FM03]
- *Arc time Petri box calculus* $atPBC$ [Nia05]
- *Generalized stochastic Petri box calculus* $gsPBC$ [MVCR08]
- *Discrete time stochastic Petri box calculus* $dtsPBC$ [Tar05,Tar06]
- *Discrete time stochastic and immediate Petri box calculus* $dtsiPBC$ [TMV10]

Syntax

The *set of all finite multisets* over X is $\mathbb{N}_f^X$.

The *set of all subsets* of X is 2^X.

$Act = \{a, b, \ldots\}$ is the set of *elementary actions*.

$\widehat{Act} = \{\hat{a}, \hat{b}, \ldots\}$ is the set of *conjugated actions (conjugates)* s.t. $a \neq \hat{a}$ and $\hat{\hat{a}} = a$.

$\mathcal{A} = Act \cup \widehat{Act}$ is the set of *all actions*.

$\mathcal{L} = \mathbb{N}_f^{\mathcal{A}}$ is the set of *all multiactions*.

The *alphabet* of $\alpha \in \mathcal{L}$ is $\mathcal{A}(\alpha) = \{x \in \mathcal{A} \mid \alpha(x) > 0\}$.

An *activity (stochastic multiaction)* is a pair (α, ρ), where
$\alpha \in \mathcal{L}$ and $\rho \in (0; 1)$ is the *probability* of multiaction α.

$\mathcal{SL}$ is the set of *all activities*.

The *alphabet* of $(\alpha, \rho) \in \mathcal{SL}$ is $\mathcal{A}(\alpha, \rho) = \mathcal{A}(\alpha)$.

The operations: *sequential execution* ;, *choice* $[]$, *parallelism* $\|$, *relabeling* $[f]$, *restriction* rs, *synchronization* sy and *iteration* $[**]$.

Sequential execution and choice have the standard interpretation.

Parallelism does not include synchronization unlike that in standard process algebras.

Relabeling functions $f : \mathcal{A} \to \mathcal{A}$ are bijections preserving conjugates: $\forall x \in \mathcal{A}\ f(\hat{x}) = \widehat{f(x)}$.
For $\alpha \in \mathcal{L}$, let $f(\alpha) = \sum_{x\in\alpha} f(x)$.

Restriction over an action a: any process behaviour containing a or its conjugate $\hat{a}$ is not allowed.

Let $\alpha, \beta \in \mathcal{L}$ be two multiactions s.t. for $a \in Act$ we have $a \in \alpha$ and $\hat{a} \in \beta$ or $\hat{a} \in \alpha$ and $a \in \beta$.
Synchronization of α and β by a is $\alpha \oplus_a \beta = \gamma$:

$$\gamma(x) = \begin{cases} \alpha(x) + \beta(x) - 1, & x = a \text{ or } x = \hat{a}; \\ \alpha(x) + \beta(x), & \text{otherwise}. \end{cases}$$

In the iteration, the initialization subprocess is executed first,
then the body one is performed zero or more times, finally, the termination one is executed.

Static expressions specify the structure of processes.

Definition 1 *Let* $(\alpha,\rho)\in\mathcal{SL}$ *and* $a\in Act$. *A* static expression *of* $dtsPBC$ *is*

$$E ::= \ (\alpha,\rho) \mid E;E \mid E[]E \mid E\|E \mid E[f] \mid E \text{ rs } a \mid E \text{ sy } a \mid [E*E*E].$$

$StatExpr$ is the set of *all static expressions* of $dtsPBC$.

Definition 2 *Let* $(\alpha,\rho)\in\mathcal{SL}$ *and* $a\in Act$. *A* regular static expression *of* $dtsPBC$ *is*

$$E ::= \ (\alpha,\rho) \mid E;E \mid E[]E \mid E\|E \mid E[f] \mid E \text{ rs } a \mid E \text{ sy } a \mid [E*D*E],$$

where $D ::= \ (\alpha,\rho) \mid D;E \mid D[]D \mid D[f] \mid D \text{ rs } a \mid D \text{ sy } a \mid [D*D*E].$

$RegStatExpr$ is the set of *all regular static expressions* of $dtsPBC$.

Dynamic expressions specify the states of processes.

Dynamic expressions are combined from static ones annotated with upper or lower bars.

The *underlying static expression* of a dynamic one: removing all upper and lower bars.

Definition 3 *Let $E \in StatExpr$ and $a \in Act$. A dynamic expression of $dtsPBC$ is*

$$G ::= \overline{E} \mid \underline{E} \mid G;E \mid E;G \mid G[]E \mid E[]G \mid G\|G \mid G[f] \mid G \text{ rs } a \mid G \text{ sy } a \mid [G{*}E{*}E] \mid [E{*}G{*}E] \mid [E{*}E{*}G].$$

$DynExpr$ is the set of *all dynamic expressions* of $dtsPBC$.

A *regular dynamic expression*: its underlying static expression is regular.

$RegDynExpr$ is the set of *all regular dynamic expressions* of $dtsPBC$.

Operational semantics

Inaction rules

Inaction rules: instantaneous structural transformations.

Let $E, F, K \in RegStatExpr$ and $a \in Act$.

Inaction rules for overlined and underlined regular static expressions

$\overline{E;F} \Rightarrow \overline{E};F$	$\underline{E};F \Rightarrow E;\overline{F}$	$E;\underline{F} \Rightarrow \underline{E;F}$
$\overline{E[]F} \Rightarrow \overline{E}[]F$	$\overline{E[]F} \Rightarrow E[]\overline{F}$	$\underline{E}[]F \Rightarrow \underline{E[]F}$
$E[]\underline{F} \Rightarrow \underline{E[]F}$	$\overline{E\|F} \Rightarrow \overline{E}\|\overline{F}$	$\underline{E}\|\underline{F} \Rightarrow \underline{E\|F}$
$\overline{E[f]} \Rightarrow \overline{E}[f]$	$\underline{E}[f] \Rightarrow \underline{E[f]}$	$\overline{E\ \mathsf{rs}\ a} \Rightarrow \overline{E}\ \mathsf{rs}\ a$
$\underline{E}\ \mathsf{rs}\ a \Rightarrow \underline{E\ \mathsf{rs}\ a}$	$\overline{E\ \mathsf{sy}\ a} \Rightarrow \overline{E}\ \mathsf{sy}\ a$	$\underline{E}\ \mathsf{sy}\ a \Rightarrow \underline{E\ \mathsf{sy}\ a}$
$\overline{[E*F*K]} \Rightarrow [\overline{E}*F*K]$	$[\underline{E}*F*K] \Rightarrow [E*\overline{F}*K]$	$[E*\underline{F}*K] \Rightarrow [E*\overline{F}*K]$
$[E*\underline{F}*K] \Rightarrow [E*F*\overline{K}]$	$[E*F*\underline{K}] \Rightarrow \underline{[E*F*K]}$	

Let $E, F \in RegStatExpr,\ G, H, \widetilde{G}, \widetilde{H} \in RegDynExpr$ and $a \in Act$.

Inaction rules for arbitrary regular dynamic expressions

$$\frac{G\Rightarrow\widetilde{G},\ \circ\in\{;,[]\}}{G\circ E\Rightarrow\widetilde{G}\circ E} \quad \frac{G\Rightarrow\widetilde{G},\ \circ\in\{;,[]\}}{E\circ G\Rightarrow E\circ\widetilde{G}} \quad \frac{G\Rightarrow\widetilde{G}}{G\|H\Rightarrow\widetilde{G}\|H} \quad \frac{H\Rightarrow\widetilde{H}}{G\|H\Rightarrow G\|\widetilde{H}} \quad \frac{G\Rightarrow\widetilde{G}}{G[f]\Rightarrow\widetilde{G}[f]}$$

$$\frac{G\Rightarrow\widetilde{G},\ \circ\in\{\mathrm{rs},\mathrm{sy}\}}{G\circ a\Rightarrow\widetilde{G}\circ a} \quad \frac{G\Rightarrow\widetilde{G}}{[G*E*F]\Rightarrow[\widetilde{G}*E*F]} \quad \frac{G\Rightarrow\widetilde{G}}{[E*G*F]\Rightarrow[E*\widetilde{G}*F]} \quad \frac{G\Rightarrow\widetilde{G}}{[E*F*G]\Rightarrow[E*F*\widetilde{G}]}$$

An *operative regular dynamic expression* G: no inaction rule can be applied to it.

$OpRegDynExpr$ is the set of *all operative regular dynamic expressions* of $dtsPBC$.

We shall consider regular expressions only and omit the word “regular”.

Definition 4 *$\approx\ = (\Rightarrow \cup \Leftarrow)^*$ is the structural equivalence of dynamic expressions in $dtsPBC$. G and G' are structurally equivalent, $G\approx G'$, if they can be reached each from other by applying inaction rules in forward or backward direction.*

Action and empty loop rules

Action rules: execution of non-empty multisets of activities at a time step.

Empty loop rule: execution of the empty multiset of activities at a time step.

For $\Gamma \in I\!N_f^{\mathcal{SL}}$, let $f(\Gamma) = \sum_{(\alpha,\rho)\in\Gamma}(f(\alpha),\rho)$.

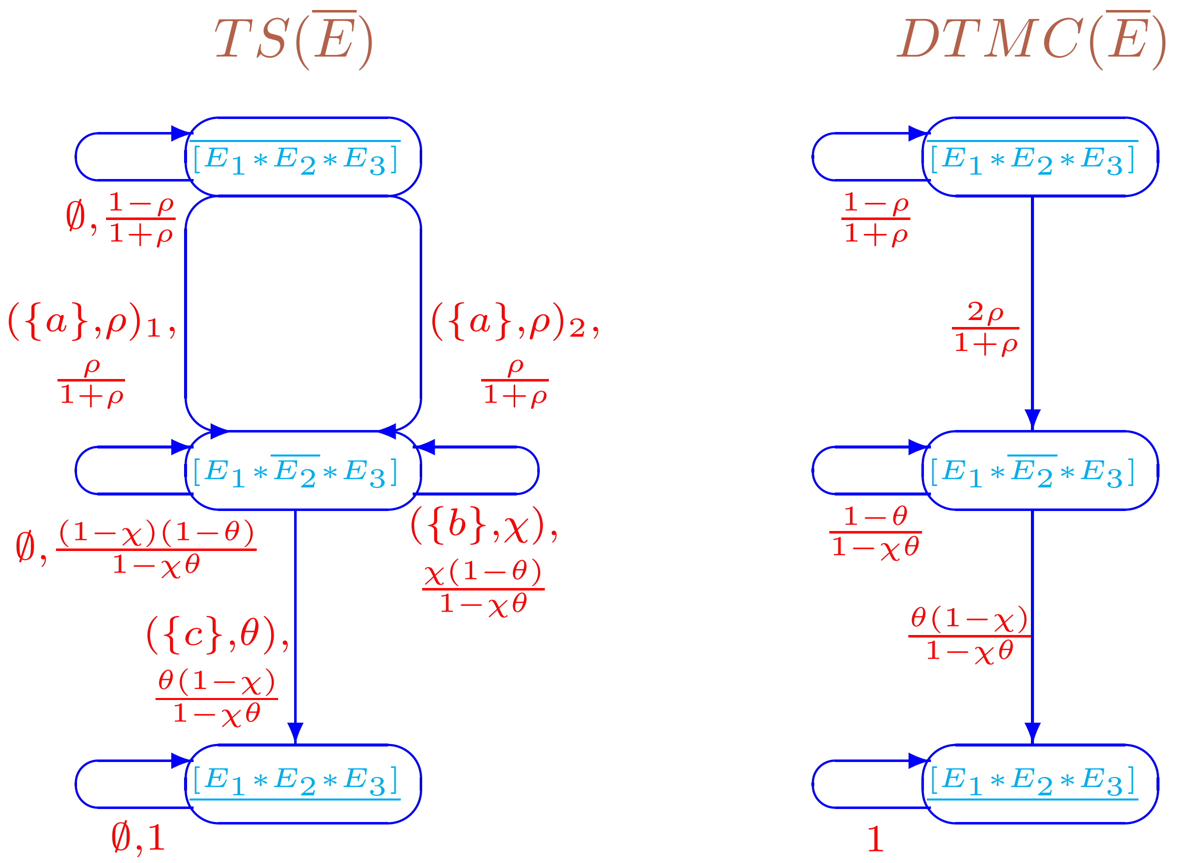
The *alphabet* of $\Gamma \in I\!N_f^{\mathcal{SL}}$ is $\mathcal{A}(\Gamma) = \cup_{(\alpha,\rho)\in\Gamma}\mathcal{A}(\alpha)$.

Let $(\alpha,\rho),(\beta,\chi) \in \mathcal{SL},\ E,F \in RegStatExpr,\ G,H \in OpRegDynExpr,$ $\widetilde{G},\widetilde{H} \in RegDynExpr,\ a \in Act$ and $\Gamma,\Delta \in I\!N_f^{\mathcal{SL}} \setminus \{\emptyset\},\ \Gamma' \in I\!N_f^{\mathcal{SL}}$.

Action and empty loop rules

E $G \stackrel{\emptyset}{\rightarrow} G$	**B** $\overline{(\alpha,\rho)} \stackrel{\{(\alpha,\rho)\}}{\longrightarrow} \underline{(\alpha,\rho)}$	**SC1** $\dfrac{G \stackrel{\Gamma}{\rightarrow} \widetilde{G},\ \circ\in\{;,[]\}}{G\circ E \stackrel{\Gamma}{\rightarrow} \widetilde{G}\circ E}$
SC2 $\dfrac{G \stackrel{\Gamma}{\rightarrow} \widetilde{G},\ \circ\in\{;,[]\}}{E\circ G \stackrel{\Gamma}{\rightarrow} E\circ\widetilde{G}}$	**P1** $\dfrac{G \stackrel{\Gamma}{\rightarrow} \widetilde{G}}{G\|H \stackrel{\Gamma}{\rightarrow} \widetilde{G}\|H}$	**P2** $\dfrac{H \stackrel{\Gamma}{\rightarrow} \widetilde{H}}{G\|H \stackrel{\Gamma}{\rightarrow} G\|\widetilde{H}}$
P3 $\dfrac{G \stackrel{\Gamma}{\rightarrow} \widetilde{G},\ H \stackrel{\Delta}{\rightarrow} \widetilde{H}}{G\|H \stackrel{\Gamma+\Delta}{\longrightarrow} \widetilde{G}\|\widetilde{H}}$	**L** $\dfrac{G \stackrel{\Gamma}{\rightarrow} \widetilde{G}}{G[f] \stackrel{f(\Gamma)}{\longrightarrow} \widetilde{G}[f]}$	**Rs** $\dfrac{G \stackrel{\Gamma}{\rightarrow} \widetilde{G},\ a,\hat{a}\not\in\mathcal{A}(\Gamma)}{G \text{ rs } a \stackrel{\Gamma}{\rightarrow} \widetilde{G} \text{ rs } a}$
I1 $\dfrac{G \stackrel{\Gamma}{\rightarrow} \widetilde{G}}{[G*E*F] \stackrel{\Gamma}{\rightarrow} [\widetilde{G}*E*F]}$	**I2** $\dfrac{G \stackrel{\Gamma}{\rightarrow} \widetilde{G}}{[E*G*F] \stackrel{\Gamma}{\rightarrow} [E*\widetilde{G}*F]}$	**I3** $\dfrac{G \stackrel{\Gamma}{\rightarrow} \widetilde{G}}{[E*F*G] \stackrel{\Gamma}{\rightarrow} [E*F*\widetilde{G}]}$
Sy1 $\dfrac{G \stackrel{\Gamma}{\rightarrow} \widetilde{G}}{G \text{ sy } a \stackrel{\Gamma}{\rightarrow} \widetilde{G} \text{ sy } a}$	**Sy2** $\dfrac{G \text{ sy } a \xrightarrow{\Gamma'+\{(\alpha,\rho)\}+\{(\beta,\chi)\}} \widetilde{G} \text{ sy } a,\ a\in\alpha,\ \hat{a}\in\beta}{G \text{ sy } a \xrightarrow{\Gamma'+\{(\alpha\oplus_a\beta,\rho\cdot\chi)\}} \widetilde{G} \text{ sy } a}$	

Transition systems



EXPRIT:The transition system and the underlying DTMC of $\overline{E}$ for $E = [((\{a\},\rho)_1[](\{a\},\rho)_2) * (\{b\},\chi) * (\{c\},\theta)]$

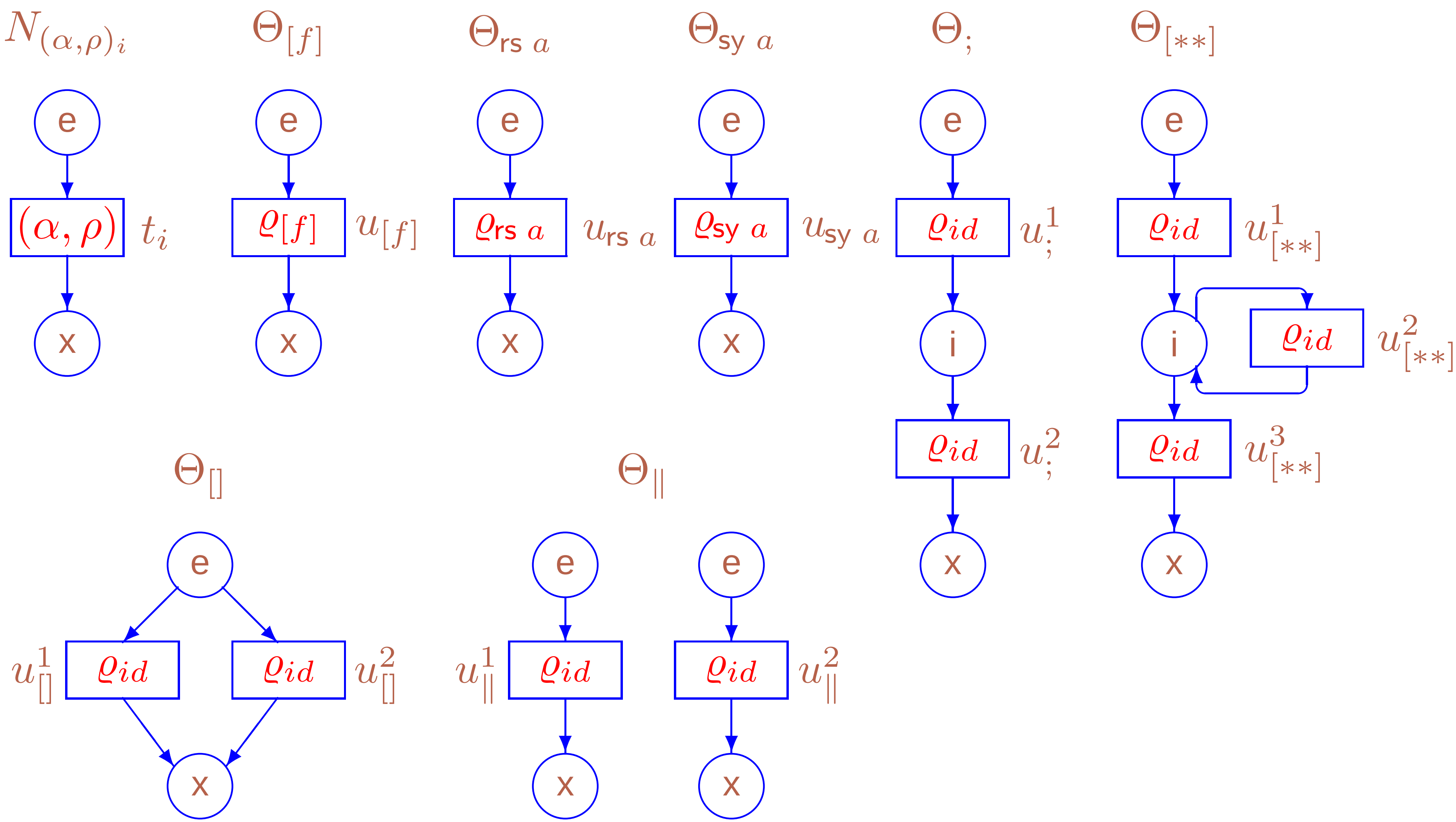
Let $E_1 = (\{a\},\rho)[](\{a\},\rho),\ E_2 = (\{b\},\chi),\ E_3 = (\{c\},\theta)$ and $E = [E_1 * E_2 * E_3]$.

The identical activities of the composite static expression are enumerated as:

$E = [((\{a\},\rho)_1[](\{a\},\rho)_2) * (\{b\},\chi) * (\{c\},\theta)]$. The derivation set $DR(\overline{E})$ of $\overline{E}$ consists of

$s_1 = [\overline{[E_1 * E_2 * E_3]}]_\approx,\ s_2 = [[E_1 * \overline{E_2} * E_3]]_\approx,\ s_3 = [\underline{[E_1 * E_2 * E_3]}]_\approx$.

Denotational semantics



The plain and operator dts-boxes

Definition 5 *Let* $(\alpha, \rho) \in \mathcal{SL},\ a \in Act$ *and* $E, F, K \in RegStatExpr$. *The* denotational semantics *of* $dtsPBC$ *is a mapping* Box_{dts} *from* $RegStatExpr$ *into plain dts-boxes:*

1. $Box_{dts}((\alpha, \rho)_i) = N_{(\alpha,\rho)_i}$;
2. $Box_{dts}(E \circ F) = \Theta_{\circ}(Box_{dts}(E), Box_{dts}(F)),\ \circ \in \{;, [], \|\}$;
3. $Box_{dts}(E[f]) = \Theta_{[f]}(Box_{dts}(E))$;
4. $Box_{dts}(E \circ a) = \Theta_{\circ a}(Box_{dts}(E)),\ \circ \in \{\mathrm{rs}, \mathrm{sy}\}$;
5. $Box_{dts}([E * F * K]) = \Theta_{[**]}(Box_{dts}(E), Box_{dts}(F), Box_{dts}(K))$.

For $E \in RegStatExpr$, let $Box_{dts}(\overline{E}) = \overline{Box_{dts}(E)}$ and $Box_{dts}(\underline{E}) = \underline{Box_{dts}(E)}$.

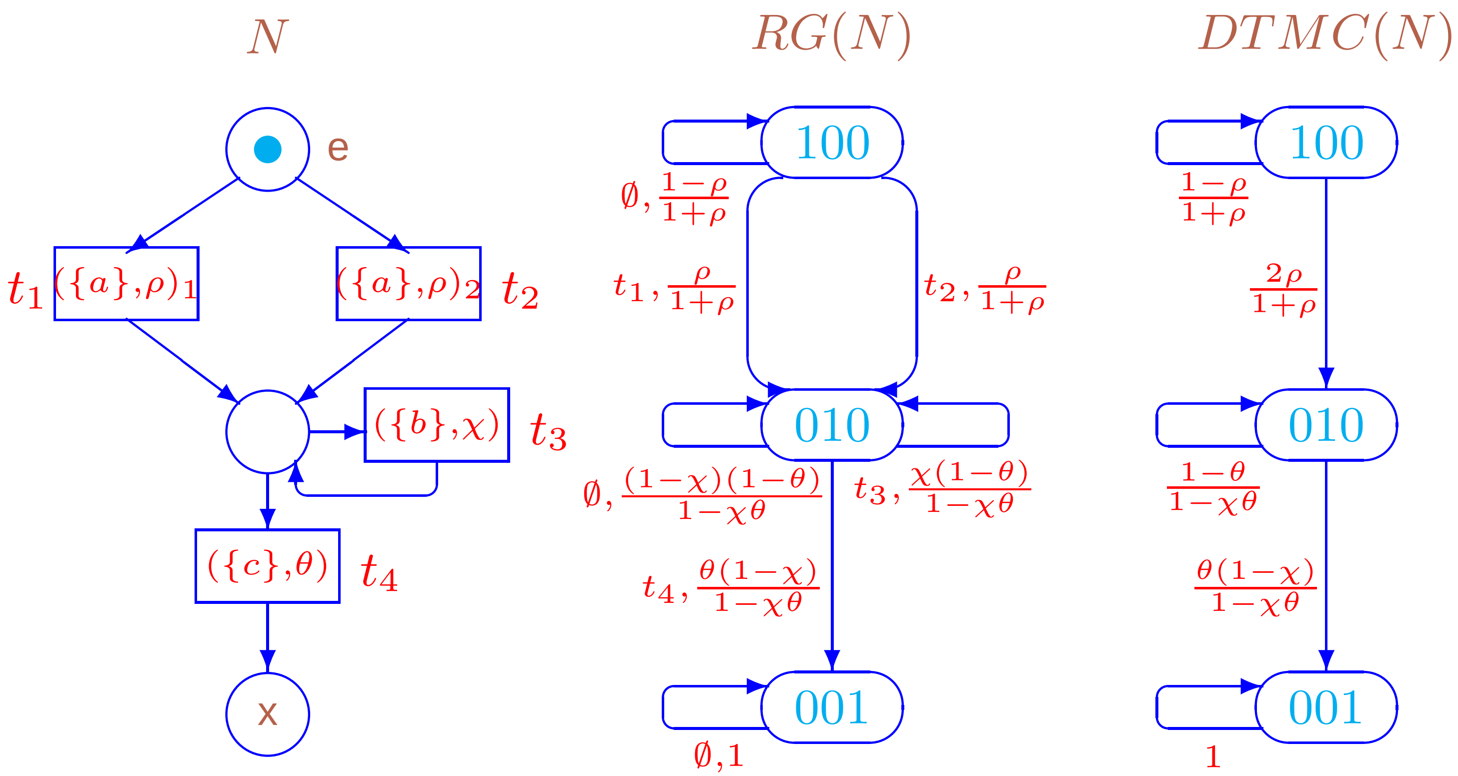
We denote isomorphism of transition systems by $\simeq$,

and the same symbol denotes isomorphism of reachability graphs and DTMCs

as well as isomorphism between transition systems and reachability graphs.

Theorem 1 *For any static expression* E *we have* $TS(\overline{E}) \simeq RG(Box_{dts}(\overline{E}))$.

Proposition 1 *For any static expression* E *we have* $DTMC(\overline{E}) \simeq DTMC(Box_{dts}(\overline{E}))$.



BOXIT:The marked dts-box $N = Box_{dts}(\overline{E})$ for $E = [((\{a\}, \rho)_1[](\{a\}, \rho)_2) * (\{b\}, \chi) * (\{c\}, \theta)]$, its reachability graph and the underlying DTMC

Performance evaluation

The elements $\mathcal{P}_{ij}$ $(1 \leq i, j \leq n = |DR(G)|)$ of *(one-step) transition probability matrix (TPM)* $\mathbf{P}$ for $DTMC(G)$:

$$\mathcal{P}_{ij} = \begin{cases} PM(s_i, s_j), & s_i \to s_j; \\ 0, & \text{otherwise.} \end{cases}$$

The *transient (k-step, $k \in \mathbb{N}$) probability mass function (PMF)* $\psi[k] = (\psi_1[k], \ldots, \psi_n[k])$ for $DTMC(G)$ is the solution of $\psi[k] = \psi[0]\mathbf{P}^k$,

where $\psi[0] = (\psi_1[0], \ldots, \psi_n[0])$ is the *initial PMF*: $\psi_i[0] = \begin{cases} 1, & s_i = [G]_\approx; \\ 0, & \text{otherwise.} \end{cases}$

We have $\psi[k+1] = \psi[k]\mathbf{P},\ k \in \mathbb{N}$.

The *steady-state PMF* $\psi = (\psi_1, \ldots, \psi_n)$ for $DTMC(G)$ is the solution of $\begin{cases} \psi(\mathbf{P} - \mathbf{E}) = \mathbf{0} \\ \psi\mathbf{1}^T = 1 \end{cases}$,

where $\mathbf{0}$ is a vector with n values 0, $\mathbf{1}$ is that with n values 1.

When $DTMC(G)$ has the single steady state, $\psi = \lim_{k\to\infty} \psi[k]$.

For $s \in DR(G)$ with $s = s_i\ (1 \leq i \leq n)$ we define $\psi[k](s) = \psi_i[k]\ (k \in \mathbb{N})$ and $\psi(s) = \psi_i$.

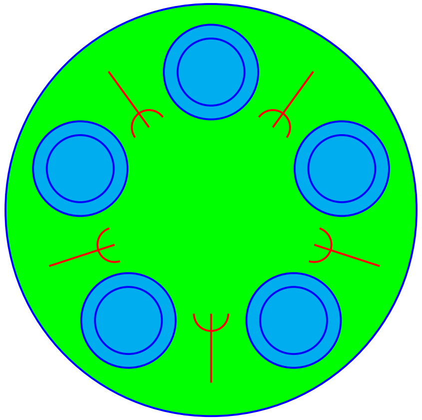
Let G be a dynamic expression and $s, \tilde{s} \in DR(G),\ S, \widetilde{S} \subseteq DR(G)$.

The following performance indices (measures) are based on the steady-state PMF.

- The *average recurrence (return) time in the state* s (the number of discrete time units or steps required for this) is $\frac{1}{\psi(s)}$.
- The *fraction of residence time in the state* s is $\psi(s)$.
- The *fraction of residence time in the set of states* $S \subseteq DR(G)$ or the *probability of the event determined by a condition that is true for all states from* S is $\sum_{s \in S} \psi(s)$.
- The *relative fraction of residence time in the set of states* S *w.r.t. that in* $\widetilde{S}$ is $\frac{\sum_{s \in S} \psi(s)}{\sum_{\tilde{s} \in \widetilde{S}} \psi(\tilde{s})}$.
- The *steady-state probability to perform a step with an activity* (α, ρ) is $\sum_{s \in DR(G)} \psi(s) \sum_{\{\Gamma \mid (\alpha,\rho) \in \Gamma\}} PT(\Gamma, s)$.
- The *probability of the event determined by a reward function* r *on the states* is $\sum_{s \in DR(G)} \psi(s) r(s)$.

Dining philosophers system

A model of five dining philosophers [P81]



The diagram of the dining philosophers system

After activation of the system, five forks appear on the table.

If the left and right forks available for a philosopher, he takes them simultaneously and begins eating.

At the end of eating, the philosopher places both his forks simultaneously back on the table.

a corresponds to the system activation.

b_i and e_i correspond to the beginning and the end of eating of philosopher i $(1 \leq i \leq 5)$.

The other actions are used for communication purpose only.

The expression of each philosopher includes two alternative subexpressions:

the second one specifies a resource (fork) sharing with the right neighbor.

The static expression of the philosopher i $(1 \leq i \leq 4)$ is

$E_i = [(\{x_i\}, \frac{1}{2}) * (((\{b_i, \widehat{y_i}\}, \frac{1}{2}); (\{e_i, \widehat{z_i}\}, \frac{1}{2}))[]((\{y_{i+1}\}, \frac{1}{2}); (\{z_{i+1}\}, \frac{1}{2}))) * \mathsf{Stop}]$.

The static expression of the philosopher 5 is

$E_5 = [(\{a, \widehat{x_1}, \widehat{x_2}, \widehat{x_2}, \widehat{x_4}\}, \frac{1}{2}) * (((\{b_5, \widehat{y_5}\}, \frac{1}{2}); (\{e_5, \widehat{z_5}\}, \frac{1}{2}))[]((\{y_1\}, \frac{1}{2}); (\{z_1\}, \frac{1}{2}))) * \mathsf{Stop}]$.

The static expression of the dining philosophers system is

$E = (E_1 \| E_2 \| E_3 \| E_4 \| E_5)$ sy x_1 sy x_2 sy x_3 sy x_4 sy y_1 sy y_2 sy y_3 sy y_4 sy y_5 sy z_1 sy z_2 sy z_3 sy z_4 sy z_5 rs x_1 rs x_2 rs x_3 rs x_4 rs y_1 rs y_2 rs y_3 rs y_4 rs y_5 rs z_1 rs z_2 rs z_3 rs z_4 rs z_5.

Interpretation of the states

s_1: the initial state,

s_2: the system is activated and no philosophers dine,

s_3: philosopher 1 dines,

s_4: philosophers 1 and 4 dine,

s_5: philosophers 1 and 3 dine,

s_6: philosopher 4 dines,

s_7: philosopher 3 dines,

s_8: philosophers 2 and 4 dine,

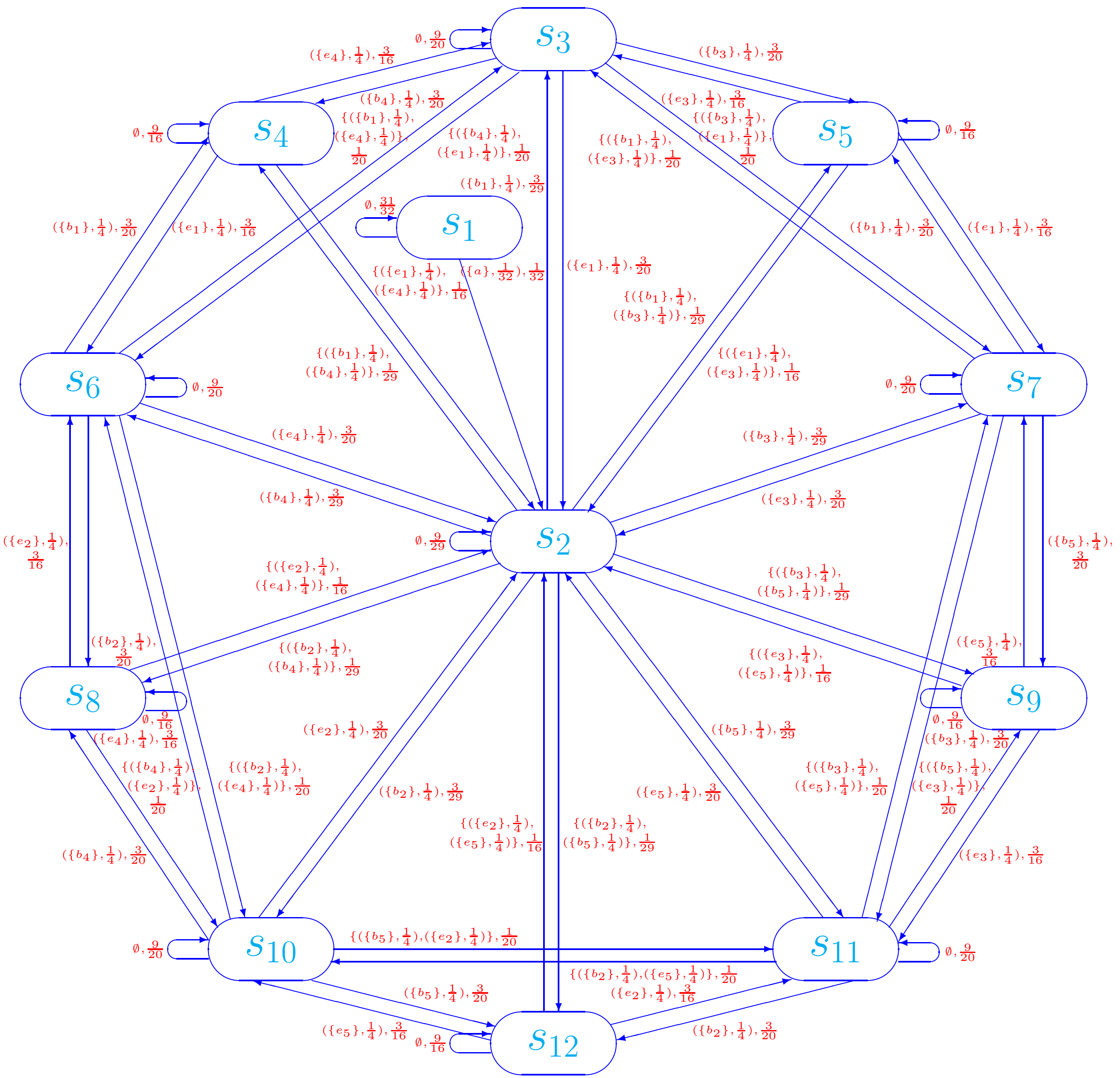
s_9: philosophers 3 and 5 dine,

s_{10}: philosopher 2 dines,

s_{11}: philosopher 5 dine,

s_{12}: philosophers 2 and 5 dine.

$TS(\overline{E})$



The transition system of the dining philosophers system

The TPM for $DTMC(\overline{E})$ is

$$\mathbf{P} = \begin{bmatrix}
\frac{31}{32} & \frac{1}{32} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{9}{29} & \frac{3}{29} & \frac{1}{29} & \frac{1}{29} & \frac{3}{29} & \frac{3}{29} & \frac{1}{29} & \frac{1}{29} & \frac{3}{29} & \frac{3}{29} & \frac{1}{29} \\
0 & \frac{3}{20} & \frac{9}{20} & \frac{3}{20} & \frac{3}{20} & \frac{1}{20} & \frac{1}{20} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{16} & \frac{3}{16} & \frac{9}{16} & 0 & \frac{3}{16} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{16} & \frac{3}{16} & 0 & \frac{9}{16} & 0 & \frac{3}{16} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{3}{20} & \frac{1}{20} & \frac{3}{20} & 0 & \frac{9}{20} & 0 & \frac{3}{20} & 0 & \frac{1}{20} & 0 & 0 \\
0 & \frac{3}{20} & \frac{1}{20} & 0 & \frac{3}{20} & 0 & \frac{9}{20} & 0 & \frac{3}{20} & 0 & \frac{1}{20} & 0 \\
0 & \frac{1}{16} & 0 & 0 & 0 & \frac{3}{16} & 0 & \frac{9}{16} & 0 & \frac{3}{16} & 0 & 0 \\
0 & \frac{1}{16} & 0 & 0 & 0 & 0 & \frac{3}{16} & 0 & \frac{9}{16} & 0 & \frac{3}{16} & 0 \\
0 & \frac{3}{20} & 0 & 0 & 0 & \frac{1}{20} & 0 & \frac{3}{20} & 0 & \frac{9}{20} & \frac{1}{20} & \frac{3}{20} \\
0 & \frac{3}{20} & 0 & 0 & 0 & 0 & \frac{1}{20} & 0 & \frac{3}{20} & \frac{1}{20} & \frac{9}{20} & \frac{3}{20} \\
0 & \frac{1}{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{16} & \frac{3}{16} & \frac{9}{16}
\end{bmatrix}.$$

The average sojourn time vector of $\overline{E}$ is

$$SJ = \left(32, \frac{29}{20}, \frac{20}{11}, \frac{16}{7}, \frac{16}{7}, \frac{20}{11}, \frac{20}{11}, \frac{16}{7}, \frac{16}{7}, \frac{20}{11}, \frac{20}{11}, \frac{16}{7}\right).$$

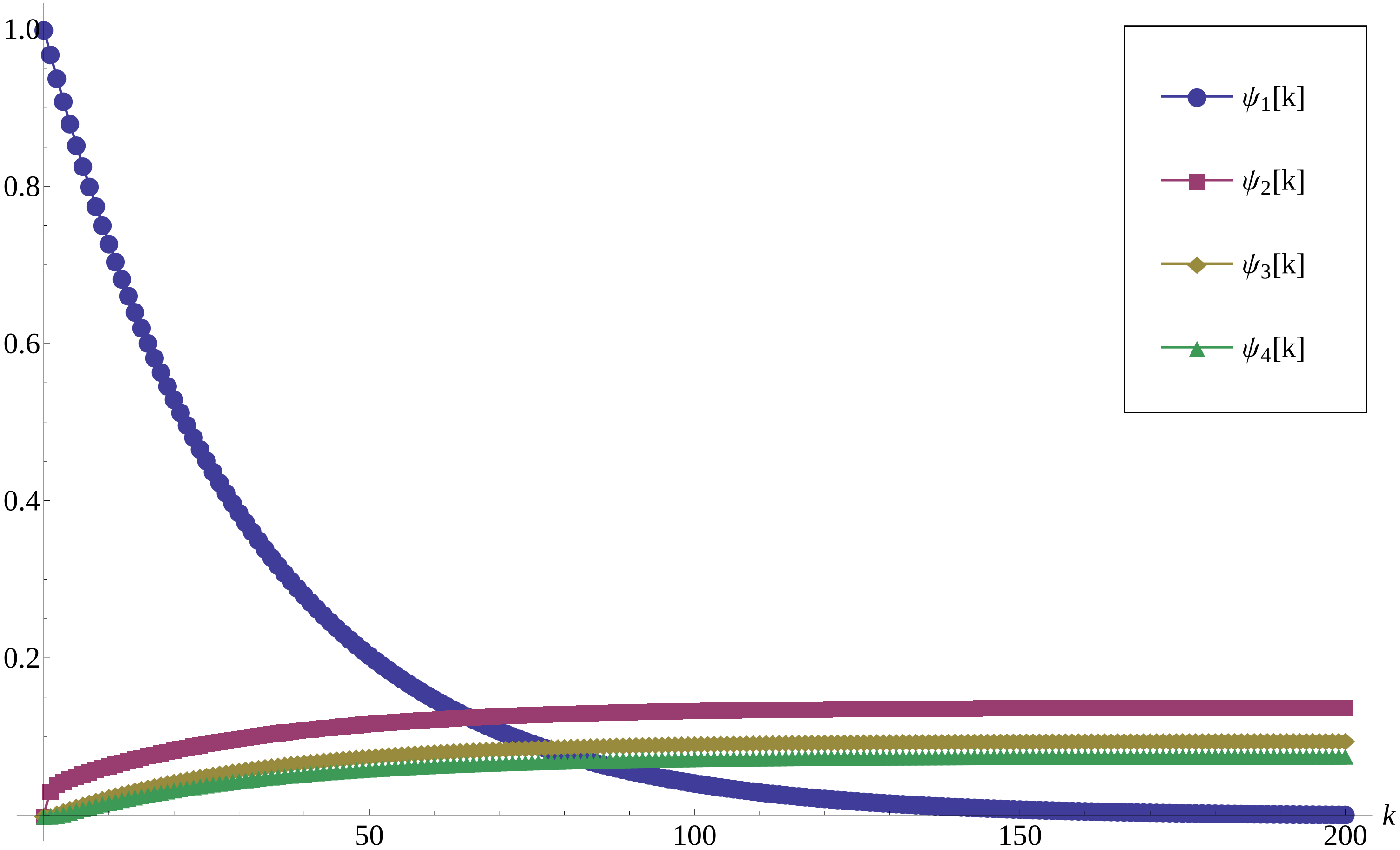
The sojourn time variance vector of $\overline{E}$ is

$$VAR = \left(1024, \frac{841}{400}, \frac{400}{121}, \frac{256}{49}, \frac{256}{49}, \frac{400}{121}, \frac{400}{121}, \frac{256}{49}, \frac{256}{49}, \frac{400}{121}, \frac{400}{121}, \frac{256}{49}\right).$$

Transient and steady-state probabilities of the dining philosophers system

k	0	20	20	60	80	100	120	140	160	180	200	∞
$\psi_1[k]$	1	0.5299	0.2808	0.1488	0.0789	0.0418	0.0222	0.0117	0.0062	0.0033	0.0017	0
$\psi_2[k]$	0	0.0842	0.1098	0.1234	0.1306	0.1345	0.1365	0.1375	0.1381	0.1384	0.1386	0.1388
$\psi_3[k]$	0	0.0437	0.0681	0.0811	0.0880	0.0916	0.0935	0.0945	0.0951	0.0954	0.0955	0.0957
$\psi_4[k]$	0	0.0335	0.0537	0.0645	0.0701	0.0732	0.0748	0.0756	0.0760	0.0763	0.0764	0.0766

We depict the probabilities for the states $s_1, \ldots, s_4$ only, since the corresponding values coincide for the states $s_3, s_6, s_7, s_{10}, s_{11}$ as well as for $s_4, s_5, s_8, s_9, s_{12}$.



Transient probabilities alteration diagram of the dining philosophers system

The steady-state PMF for $DTMC(\overline{E})$ is

$$\psi = \left(0, \frac{29}{209}, \frac{20}{209}, \frac{16}{209}, \frac{16}{209}, \frac{20}{209}, \frac{20}{209}, \frac{16}{209}, \frac{16}{209}, \frac{20}{209}, \frac{20}{209}, \frac{16}{209}\right).$$

Performance indices

- The average recurrence time in the state s_2, where all the forks are available, the *average system run-through*, is $\frac{1}{\psi_2} = \frac{209}{29} = 7\frac{6}{29}$.

- Nobody eats in the state s_2. The *fraction of time when no philosophers dine* is $\psi_2 = \frac{29}{209}$.

  Only one philosopher eats in the states $s_3, s_6, s_7, s_{10}, s_{11}$. The *fraction of time when only one philosopher dines* is $\psi_3 + \psi_6 + \psi_7 + \psi_{10} + \psi_{11} = \frac{20}{209} + \frac{20}{209} + \frac{20}{209} + \frac{20}{209} + \frac{20}{209} = \frac{100}{209}$.

  Two philosophers eat together in the states $s_4, s_5, s_8, s_9, s_{12}$. The *fraction of time when two philosophers dine* is $\psi_4 + \psi_5 + \psi_8 + \psi_9 + \psi_{12} = \frac{16}{209} + \frac{16}{209} + \frac{16}{209} + \frac{16}{209} + \frac{16}{209} = \frac{80}{209}$.

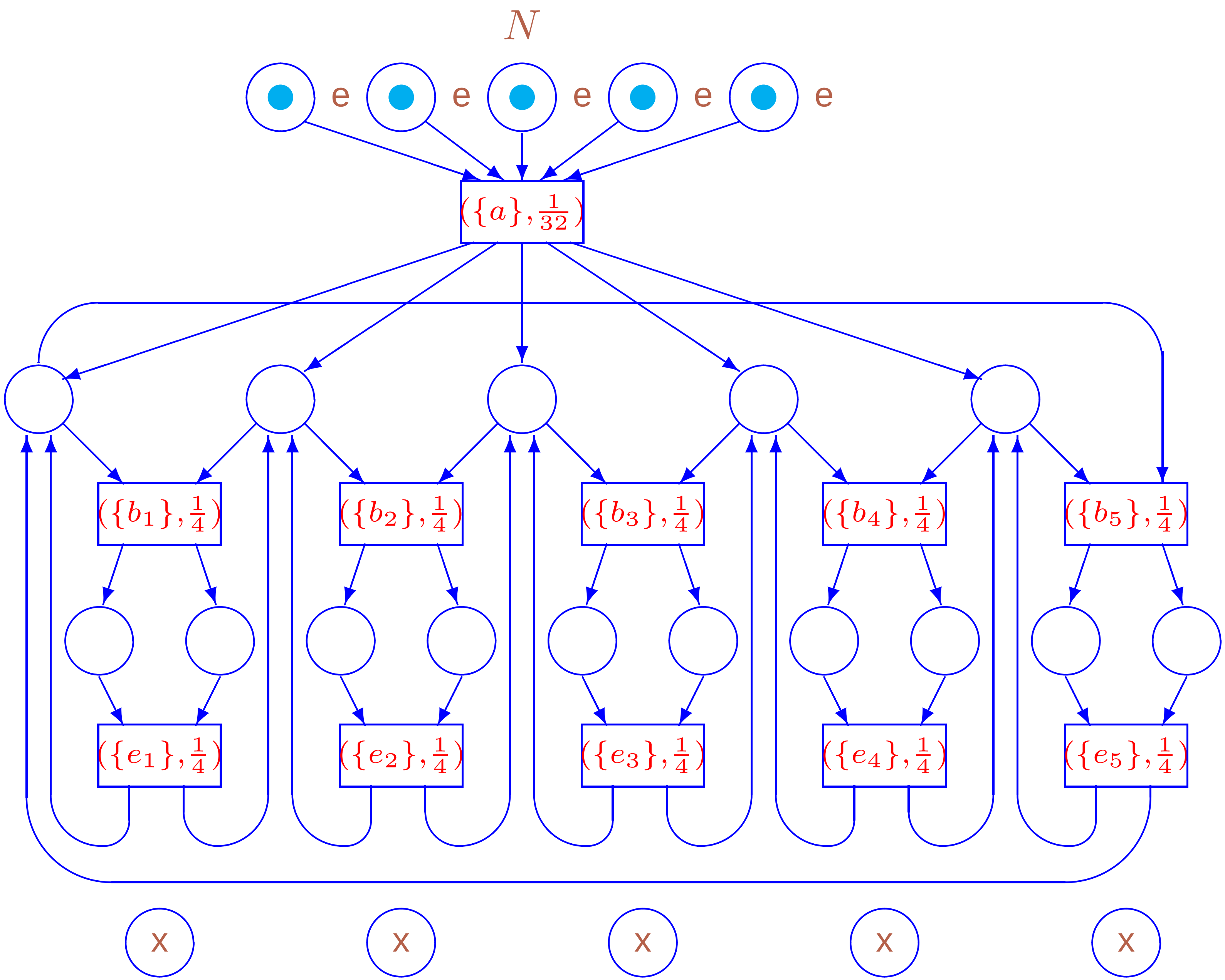
  The *relative fraction of time when two philosophers dine w.r.t. when only one philosopher dines* is $\frac{80}{209} \cdot \frac{209}{100} = \frac{4}{5}$.

- The beginning of eating of first philosopher $(\{b_1\}, \frac{1}{4})$ is only possible from the states s_2, s_6, s_7.

  The beginning of eating probability in each of the states is a sum of execution probabilities for all multisets of activities containing $(\{b_1\}, \frac{1}{4})$.

  The *steady-state probability of the beginning of eating of first philosopher* is

  $$\psi_2 \sum_{\{\Gamma|(\{b_1\},\frac{1}{4})\in\Gamma\}} PT(\Gamma, s_2) + \psi_6 \sum_{\{\Gamma|(\{b_1\},\frac{1}{4})\in\Gamma\}} PT(\Gamma, s_6) + \psi_7 \sum_{\{\Gamma|(\{b_1\},\frac{1}{4})\in\Gamma\}} PT(\Gamma, s_7) =$$
  $$\frac{29}{209}\left(\frac{3}{29} + \frac{1}{29} + \frac{1}{29}\right) + \frac{20}{209}\left(\frac{3}{20} + \frac{1}{20}\right) + \frac{20}{209}\left(\frac{3}{20} + \frac{1}{20}\right) = \frac{13}{209}.$$



The marked dts-box of the dining philosophers system

Overview and open questions

The results obtained

- A discrete time stochastic extension $dtsPBC$ of finite PBC enriched with iteration.
- A case study of performance analysis: the dining philosophers system.

Further research

- Defining stochastic equivalences to identify stochastic processes with similar behaviour.
- Extending the syntax with recursion operator.

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