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1 An overview of the results obtained

1.1 Petri nets without silent transitions

Petri nets [15] are a popular formal model for design of concurrent and distributed systems. The net itself reflects the topological structure of the system, whreas the variability of the markings represents the dynamic behavior. One of the main advantages of Petri nets is their ability for structural characterization of three fundamental features of concurrent computations: causality, nondeterminism and concurrency.

The notion of equivalence is central to any theory of systems. Equivalences allow one to compare and reduce systems taking into account particular aspects of their behaviour. In recent years, a wide range of behavioural equivalences was proposed in the concurrency theory. A lot of them were either directly defined or transferred from other formal models into the framework of Petri nets.

The equivalences can be classified depending of semantics of concurrency they impose. In *interleaving* semantics, a concurrent happening of actions is interpreted as their occurrence in any possible order. In *step* semantics, a concurrency of actions is a basic notion, but their causal dependencies are not respected. In *partial word* semantics, causal dependencies of actions are respected in part via partially ordered multisets (pomsets) of actions, and a pomset may be modelled by a less sequential one (i.e. having less strict partial order). In *pomset* semantics, causal dependencies of actions are fully respected, and pomsets of actions should coincide to model each other. In *process* semantics, a structure of a process (causal) net is respected.

The following basic notions of τ-equivalences are known from the literature.

- *Trace equivalences* (they respect only protocols of behaviour of systems): interleaving ($\equiv_i$) [10], step ($\equiv_s$) [16], partial word ($\equiv_{pw}^{\tau}$) [22] and pomset ($\equiv_{pom}$) [9].
- *Usual bisimulation equivalences* (they respect branching structure of behaviour of systems): interleaving ($\underline{\leftrightarrow}_i$) [14], step ($\underline{\leftrightarrow}_s$) [13], partial word ($\underline{\leftrightarrow}_{pw}$) [21], pomset ($\underline{\leftrightarrow}_{pom}$) [4] and process ($\underline{\leftrightarrow}_{pr}$) [2].
- *ST-bisimulation equivalences* (they respect the duration or maximality of events in behaviour of systems): interleaving ($\underline{\leftrightarrow}_{iST}$) [9], partial word ($\underline{\leftrightarrow}_{pwST}$) [21] and pomset ($\underline{\leftrightarrow}_{pomST}$) [21].
- *History preserving bisimulation equivalences* (they respect the "history" of behaviour of systems): pomset ($\underline{\leftrightarrow}_{pomh}$) [19].
- *Conflict preserving equivalences* (they completely respect conflicts of events in systems): occurrence ($\equiv_{occ}$) [9].
- *Isomorphism* ($\simeq$) (i.e. coincidence of systems up to renaming of their components).

Another important group of equivalences are back-forth bisimulation ones which are based on the idea that a bisimulation relation should not only require systems to simulate each other behaviour in the forward direction but also when going back in the history. By now, the set of all possible back-forth equivalence notions was proposed in interleaving, step, partial word and pomset semantics. Most of them coincide with basic or with other back-forth relations. The following new notions were obtained: step back step forth ($\underline{\leftrightarrow}_{sbsf}$) [5], step back partial word forth ($\underline{\leftrightarrow}_{sbpwf}$) [18] and step back pomset forth ($\underline{\leftrightarrow}_{sbpomf}$) [18] bisimulation equivalences. In [20], we supplemented them by several new relations in process semantics: step back process forth ($\underline{\leftrightarrow}_{sbprf}$) and pomset back process forth ($\underline{\leftrightarrow}_{pombprf}$) bisimulation equivalences.

The third important group of equivalences are place bisimulation ones introduced in [1]. They are relations between places (instead of markings or processes). The relation on markings is obtained using the "lifting" of relation on places. The main application of place bisimulation equivalences is an effective global behaviour preserving reduction technique for Petri nets based on them. In [1], interleaving place bisimulation equivalence ($\sim_i$) was proposed. In [2], step ($\sim_s$), partial word ($\sim_{pw}$), pomset ($\sim_{pom}$) and process ($\sim_{pr}$) place bisimulation equivalences were defined. In addition, in these papers a polynomial algorithm of a net reduction modulo $\sim_i$ and $\sim_{pr}$ was proposed.

To choose most appropriate behavioral viewpoint on systems to be modeled, it is very important to have a complete set of equivalence notions in all semantics and understand their interrelations. This branch of research is usually called *comparative concurrency semantics*.

Treating equivalences for preservation by refinements allows one to decide which of them may be used for top-down design. For such a multilevel design of concurrent systems a *refinement* operator is used. After applying it, some components of the systems become having internal structure, i.e. we consider such systems on lower abstraction level as a result.

To evaluate how equivalences respect concurrency, it is actual to consider correlation of these notions on concurrency-free (sequential) nets. In addition, it allows us to simplify check of the relations for such a net subclass because of merging some of the equivalences.

Our results in this branch of research are the following.

1. We introduced a number of new basic equivalence notions (in addition to the known from the literature ones) to obtain the representatively complete set of them.

   These new relations are: partial word trace equivalence ($\equiv_{pw}$), process trace equivalence ($\equiv_{pr}$), process ST-bisimulation equivalence ($\underline{\leftrightarrow}_{prST}$), process history preserving bisimulation equivalence ($\underline{\leftrightarrow}_{prh}$) and multi event structure conflict preserving equivalence ($\equiv_{mes}$).

2. After comparing all the equivalence notions, we obtained a complete diagram of their interrelations.

   In particular, we proved that conflict preserving equivalences are stronger than all bisimulation ones. We showed that back-forth bisimulation relations are between usual and ST-ones. We also demonstrated that $\sim_{pr}$ implies $\underline{\leftrightarrow}_{prh}$ and answer the question from [1]: it is no sense to define history preserving place bisimulation equivalence. Another consequence is: the algorithm of a net reduction from [2], based on $\sim_{pr}$, preserves "histories" of the behaviour of the initial net.

3. In [3], an SM-refinement operator for Petri nets was proposed, which "replaces" their transitions by SM-nets, a subclass of state machine nets. We treat all the considered equivalence notions for preservation by SM-refinements.

   In particular, we established that all conflict preserving and ST-bisimulation equivalences are preserved by refinements whereas among trace ones only $\equiv_{pw}$, $\equiv_{pom}$ and $\equiv_{occ}$ have such a property. We proved that $\sim_{pr}$ is the only place bisimulation equivalence which is preserved by SM-refinements. Thus, this equivalence may be used for a compositional reduction of nets.

4. We obtained a diagram of interrelations of all the equivalences on sequential nets.

   A merging of most of the relations in interleaving – pomset semantics was demonstrated.

1.2 Petri nets with silent transitions

Silent transitions are transitions labelled by special *silent* action τ which represents an internal activity of a system to be modeled, and it is invisible for an external observer. It is well-known that Petri nets with silent transitions are more powerful than usual ones.

Equivalences which abstract of silent actions are called *τ-equivalences* (these are labelled by the symbol τ to distinguish them of relations not abstracting of silent actions).

The following basic notions of τ-equivalences are known from the literature.

- *τ-trace equivalences* (they respect only protocols of behavior of systems): interleaving ($\equiv_i^\tau$) [16], step ($\equiv_s^\tau$) [16], partial word ($\equiv_{pw}^\tau$) [22] and pomset ($\equiv_{pom}^\tau$) [17].

- *Usual τ-bisimulation equivalences* (they respect branching structure of behavior of systems): interleaving ($\underline{\leftrightarrow}_i^\tau$) [11], step ($\underline{\leftrightarrow}_s^\tau$) [16], partial word ($\underline{\leftrightarrow}_{pw}^\tau$) [21] and pomset ($\underline{\leftrightarrow}_{pom}^\tau$) [17].

- *ST-τ-bisimulation equivalences* (they respect the duration or maximality of events in behavior of systems): interleaving ($\underline{\leftrightarrow}_{iST}^\tau$) [21], partial word ($\underline{\leftrightarrow}_{pwST}^\tau$) [21] and pomset ($\underline{\leftrightarrow}_{pomST}^\tau$) [21].

- *History preserving τ-bisimulation equivalences* (they respect the "past" or "history" of behavior of systems): pomset ($\underline{\leftrightarrow}_{pomh}^\tau$) [6].

- *History preserving ST-τ-bisimulation equivalences* (they respect the "history" and the duration or maximality of events in behavior of systems): pomset ($\underline{\leftrightarrow}^{\tau}_{pomhST}$) [6].
- *Usual branching τ-bisimulation equivalences* (they respect branching structure of behavior of systems taking a special care for silent actions): interleav ing ($\underline{\leftrightarrow}^{\tau}_{ibr}$) [8].
- *History preserving branching τ-bisimulation equivalences* (they respect "history" and branching structure of behavior of systems taking a special care for silent actions): pomset ($\underline{\leftrightarrow}^{\tau}_{pomhbr}$) [6].
- *Isomorphism* ($\simeq$) (i.e. coincidence of systems up to renaming of their components).

Several notions of back-forth τ-bisimulation equivalences were introduced in the literature. In [12], in the framework of transition systems with silent actions, interleaving back-forth τ-bisimulation equivalence ($\underline{\leftrightarrow}^{\tau}_{ibif}$) was defined. Its coincidence with $\underline{\leftrightarrow}^{\tau}_{ibr}$ was proved.

In [18], a new notion of τ-equivalence was proposed for event structures with silent actions: pomset back pomset forth ($\underline{\leftrightarrow}^{\tau}_{pombpomf}$) τ-bisimulation equivalence. Its coincidence with ($\underline{\leftrightarrow}^{\tau}_{pomhbr}$ was proved.

Our results in this branch of research are the following.

1. We extended the set of basic notions of τ-equivalences by interleaving ST-branching τ-bisimulation one ($\underline{\leftrightarrow}^{\tau}_{iSTbr}$), pomset history preserving ST-branching τ-bisimulation one ($\underline{\leftrightarrow}^{\tau}_{pomhSTbr}$) and multi event structure one ($\equiv^{\tau}_{mes}$).

   The set of back-forth τ-equivalences was completed by 6 new notions: interleaving back step forth ($\underline{\leftrightarrow}^{\tau}_{ibsf}$), interleaving back partial word forth ($\underline{\leftrightarrow}^{\tau}_{ibpwf}$), interleaving back pomset forth ($\underline{\leftrightarrow}^{\tau}_{ibpomf}$), step back step forth ($\underline{\leftrightarrow}^{\tau}_{sbsf}$), step back partial word forth ($\underline{\leftrightarrow}^{\tau}_{sbpwf}$) and step back pomset forth ($\underline{\leftrightarrow}^{\tau}_{sbpomf}$) bisimulation equivalences.

2. After comparing all the equivalence notions, we obtained a complete diagram of their interrelations.

   In particular, we proved that $\equiv^{\tau}_{mes}$ implies only τ-trace equivalences, not bisimulation ones unlike the situation on Petri nets without silent transitions. Another r esult is that back-forth τ-bisimulation equivalences are between usual and branching ones.

3. We investigated the interrelations of all the considered τ-equivalences with equivalences which do not abstract of silent actions.

   The result was that abstraction of silent actions produces weaker equivalences, and there is more of them, due to the new interplay of branching idea and silent actions.

4. We treated all the considered τ-equivalence notions for preservation by SM-refinements.

   We showed that $\underline{\leftrightarrow}^{\tau}_{iSTbr}$, $\underline{\leftrightarrow}^{\tau}_{pomhSTbr}$ and $\equiv^{\tau}_{mes}$, i.e. all the new basic equivalences considered in this paper, are preserved by SM-refinements. Thus, we have branching and conflict preserving equivalences which may be used for multilevel design. In the literature, a stability w.r.t. SM-refinements was proved only for $\underline{\leftrightarrow}^{\tau}_{pomhST}$ in [3] and for $\underline{\leftrightarrow}^{\tau}_{iST}$ in [7]. The preservation result for other ST-τ-bisimulation equivalences was proved in [21], but it was done on event structures and an other refinement operator was used. The preservation of trace τ-equivalences was not established before. Thus, our results for $\underline{\leftrightarrow}^{\tau}_{pwST}$, $\underline{\leftrightarrow}^{\tau}_{pomST}$, $\equiv^{\tau}_{pw}$ and $\equiv^{\tau}_{pom}$ are also new.

5. We investigated the interrelations of all the τ-equivalence notions on nets without silent transitions.

   We proved that on nets without silent transitions τ-equivalences coincide with equivalence notions which do not abstract of silent actions.

6. We investigated equivalences on sequential nets.

   We demonstrated that on such nets interleaving and pomset τ-equivalences are merged, and back-forth τ-equivalences coincide with forth τ-equivalence relations.

2 List of papers produced during period of the grant

1. Tarasyuk I.V. *Place bisimulation equivalences for design of concurrent systems.* Pre-proceedings of MFCS'98 Workshop on Concurrency, Brno, Czech Republic, August 27–29, 1998, Petr Janchar and Mojímir Kretinský, eds., Faculty of Informatics, Masaryk University, *FIMU Report Series* **FIMU-RS-98-06**, p. 184–198, July 1998.

2. TARASYUK I.V. *Place bisimulation equivalences for design of concurrent and sequential systems.* Proceedings of MFCS'98 Workshop on Concurrency, Brno, Czech Republic, August 27–29, 1998, *Electronic Notes in Theoretical Computer Science* **18**, 16 p., 1998.

3. TARASYUK I.V. *τ-equivalences and refinement.* Proceedings of International Refinement Workshop and Formal Methods Pacific - 98 (IRW/FMP'98), Work-in-Progress Papers, Canberra, Australia, September 29 – October 2, 1998, Jim Grundy, Martin Schwenke and Trevor Vickers, eds., *Joint Computer Science Technical Report Series* **TR-CS-98-09**, The Australian National University, p. 110–128, September 1998.

4. TARASYUK I.V. *Basic behavioural equivalences of Petri nets for design of concurrent systems. Informatika*, Ljubljana, Slovenija, 1999 (to be published).

5. TARASYUK I.V. *τ-equivalences and refinement for Petri nets based design.* 40 p., March 1999 (to be submitted).

3 Abstracts of the papers

- The paper [4] is devoted to the investigation of behavioral equivalences of concurrent systems modeled by Petri nets. The basic equivalence notions known from the literature are supplemented by new ones and examined for all class of nets as well as for their subclass of sequential nets which are a special subclass of general Petri nets modeling sequential systems, where no two actions can happen simultaneously. The complete diagram of interrelations of the considered equivalences is obtained. In addition, the preservation of the equivalence notions by refinements is treated, which gives possibility to consider the behavior of nets on lower abstraction level.

- In the papers [1,2], we supplement the set of basic and back-forth behavioural equivalences for Petri nets by place bisimulation ones. The relationships of all the equivalence notions are examined, and their preservation by refinements is investigated to find out which of these relations may be used in top-down design. It is demonstrated that the place bisimulation equivalences may be used for the compositional and history preserving reduction of Petri nets. In addition, we consider all the mentioned equivalences on sequential nets. On this net subclass all pomset equivalences merge with the corresponding interleaving ones, and it allows one to simplify their check.

- The papers [3,5] are devoted to the investigation of behavioral equivalences of concurrent systems modeled by Petri nets with silent transitions. Basic τ-equivalences and back-forth τ-bisimulation equivalences known from the literature are supplemented by new ones, giving rise to complete set of equivalence notions in interleaving – true concurrency and linear – branching time semantics. Their interrelations are examined for the general class of nets as well as for their subclasses of nets without silent transitions and sequential nets. In addition, the preservation of all the equivalence notions by refinements is investigated.

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