Report on Postdoctoral Research Supported by DFG Stipend of Computer Science Graduate Programme at TU Dresden

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Introduction to the research area

1. Petri nets became a popular formal model for design of concurrent and distributed systems. Silent transitions are transitions labeled by special *silent* action τ which represents an internal activity of a system to be modeled and it is invisible for external observer. It is well-known that Petri nets with silent transitions are more powerful than usual ones.

   The notion of equivalence is central in any theory of systems. It allows to compare systems taking into account particular aspects of their behavior. Equivalences which abstract of silent actions are called τ-*equivalences* (these are labeled by the symbol τ to distinguish them of relations not abstracting of silent actions). In recent years, a wide range of semantic equivalences was proposed in concurrency theory, see [12, 14, 23, 26]. Some of them were either directly defined or transferred from other formal models to Petri nets. The following *basic* notions of τ-equivalences are known from the literature.

   - *τ-trace equivalences* (they respect only protocols of behavior of systems): interleaving ($\equiv_i^\tau$), step ($\equiv_s^\tau$), partial word ($\equiv_{pw}^\tau$) and pomset ($\equiv_{pom}^\tau$).
   - *Usual τ-bisimulation equivalences* (they respect branching structure of behavior of systems): interleaving ($\underline{\leftrightarrow}_i^\tau$), step ($\underline{\leftrightarrow}_s^\tau$), partial word ($\underline{\leftrightarrow}_{pw}^\tau$) and pomset ($\underline{\leftrightarrow}_{pom}^\tau$).
   - *ST-τ-bisimulation equivalences* (they respect the duration or maximality of events in behavior of systems): interleaving ($\underline{\leftrightarrow}_{iST}^\tau$), partial word ($\underline{\leftrightarrow}_{pwST}^\tau$) and pomset ($\underline{\leftrightarrow}_{pomST}^\tau$).
   - *History preserving τ-bisimulation equivalences* (they respect the "past" or "history" of behavior of systems): pomset ($\underline{\leftrightarrow}_{pomh}^\tau$).
   - *History preserving ST-τ-bisimulation equivalences* (they respect the "history" and the duration or maximality of events in behavior of systems): pomset ($\underline{\leftrightarrow}_{pomhST}^\tau$).
   - *Usual branching τ-bisimulation equivalences* (they respect branching structure of behavior of systems taking a special care for silent actions): interleaving ($\underline{\leftrightarrow}_{ibr}^\tau$).
   - *History preserving branching τ-bisimulation equivalences* (they respect "history" and branching structure of behavior of systems taking a special care for silent actions): pomset ($\underline{\leftrightarrow}_{pomhbr}^\tau$).
   - *Isomorphism* ($\simeq$) (i.e. coincidence of systems up to renaming of their components).

   Another type of equivalence notions called *back-forth* bisimulation equivalences are based on the idea that bisimulation relation do not only require systems to simulate each other behavior in the forward direction (as usually) but also when going back in history. They are closely connected with equivalences of logics with past modalities. These equivalence notions were initially introduced in [21]. In the framework of transition systems without silent actions interleaving back-forth bisimulation equivalence ($\underline{\leftrightarrow}_{ibif}$) was defined and proved to merge with $\underline{\leftrightarrow}_i$. On transition systems with silent actions it was shown that back-forth variant ($\underline{\leftrightarrow}_{ibif}^\tau$) of interleaving τ-bisimulation equivalence coincide with $\underline{\leftrightarrow}_{ibr}^\tau$. In [9, 10], the new variants of step, partial word and pomset back-forth bisimulation equivalences were defined in the framework of prime event structures without silent actions. In [22], the new idea of differentiating the kinds of back and forth simulations appeared (following this idea, it is possible, for example, to define step back pomset forth bisimulation equivalence). All possible back-forth equivalence notions were proposed in interleaving, step, partial word and pomset semantics for prime event structures without silent actions. The new notion of τ-equivalence was proposed for event structures with silent actions: pomset back pomset forth ($\underline{\leftrightarrow}^{\tau}_{pombpomf}$) τ-bisimulation equivalence. It's coincidence with $\underline{\leftrightarrow}^{\tau}_{pomhbr}$ was proved.

To choose most appropriate behavioral viewpoint on systems to be modeled, it is very important to have a complete set of equivalence notions in all semantics and understand their interrelations. To clarify the nature of equivalences and evaluate how they respect internal activity and concurrency in systems to be modeled, it is actual to consider also correlation of these notions on nets without silent transitions and concurrency-free (sequential) ones. Treating equivalences for preservation by refinements allows one to decide which of them may be used for top-down design.

2. Stochastic Petri nets (SPNs) are an established model type for the quantitative analysis of Discrete Event Dynamic Systems (DEDSs). SPNs have been proposed about twenty years ago [13, 19] and are mainly considered on a continuous time scale which usually means that exponential or phase type distributions are associated with transitions. In this way, the stochastic process underlying an SPN is a Continuous Time Markov Chain (CTMC) which can be generated and analyzed with well-known methods [24]. One particular characterization of this class of SPNs is that only single transitions fire, such that the well-known interleaving semantics is the basic approach for defining the dynamic behavior of SPNs. This interleaving behavior is also used for generalized stochastic Petri nets (GSPNs) [1, 7] which include transitions with exponential firing delay and that with zero firing delay. Even for such immediate transitions with instantaneous firing interleaving semantics is commonly considered. For SPNs and GSPNs, labeling of transitions has been introduced recently [5, 6]. After definition of transition labeling it is possible to define bisimulation equivalence for SPNs and GSPNs such that equivalent nets behave identically from a stochastic point of view. Details about the approach which introduces bisimulation for CTMCs with labeled transitions can be found in [4, 5, 15, 16].

   Apart from continuous time distributions also discrete time distributions can be assigned to transitions of Petri nets. Usually geometric distributions or mixtures of geometric distributions are used. First approaches have been published about 15 years ago [20], but also more recent extensions of the basic class of nets with discrete time steps have been proposed [27, 28]. To distinguish continuous and discrete time SPNs, we denote the former as CTSPNs and the latter as DTSPNs. DTSPNs describe an underlying Discrete Time Markov Chain (DTMC). The major problem with this model class is that transitions fire concurrently such that steps instead of interleavings have to be considered. This makes the interpretation and analysis of the model class more complex. For DTSPNs labeling of transitions and an adequate definition of equivalence has not been introduced yet.

   Recently, Prof. Dr. Peter Buchholz introduced a new class of DTSPNs with labeled transitions. Transitions can be also labeled by silent action, and in this way they can be considered as invisible for an external observer. The dynamic behavior of this class of nets is characterized by steps instead of single transitions. The underlying stochastic process is still a DTMC, however, transitions of the DTMC describe sets of transitions that fire concurrently. Thus, commonly used notions defining bisimulation or trace equivalence of probabilistic processes [11, 18] are not adequate for this type of model. No algebraic specification for such a class of stochastic processes was presented so far.

The results obtained

1. Working in the framework of Petri nets with silent transitions, we continued the research of [25] and extended the set of basic notions of τ-equivalences by interleaving ST-branching τ-bisimulation one ($\underline{\leftrightarrow}^{\tau}_{iSTbr}$), pomset history preserving ST-branching τ-bisimulation one ($\underline{\leftrightarrow}^{\tau}_{pomhSTbr}$) and multi event structure one ($\equiv^{\tau}_{mes}$). Let us note that an idea to introduce $\underline{\leftrightarrow}^{\tau}_{pomhSTbr}$ appeared initially in [22] on the model of event structures. We completed back-forth τ-equivalences from [22] by 6 new notions: interleaving back step forth ($\underline{\leftrightarrow}^{\tau}_{ibsf}$), interleaving back partial word forth ($\underline{\leftrightarrow}^{\tau}_{ibpwf}$), interleaving back pomset forth ($\underline{\leftrightarrow}^{\tau}_{ibpomf}$), step back step forth ($\underline{\leftrightarrow}^{\tau}_{sbsf}$), step back partial word forth ($\underline{\leftrightarrow}^{\tau}_{sbpwf}$) and step back pomset forth ($\underline{\leftrightarrow}^{\tau}_{sbpomf}$) τ-bisimulation equivalences. We compared all back-forth τ-equivalences with the set of basic behavioral relations. We also investigated the interrelations of all the considered τ-equivalences with equivalences which do not abstract of silent actions.

   In [3], SM-refinement operator for Petri nets was proposed, which "replaces" their transitions by SM-nets, a special subclass of state machine nets. We treated all the considered τ-equivalence notions for preservation by SM-refinements. We showed that $\underline{\leftrightarrow}^{\tau}_{iSTbr}$, $\underline{\leftrightarrow}^{\tau}_{pomhSTbr}$ and $\equiv^{\tau}_{mes}$, i.e. all the new basic equivalences considered in this paper, are preserved by SM-refinements. Thus, we have branching and conflict preserving equivalences which may be used for multilevel design. In the literature, a stability

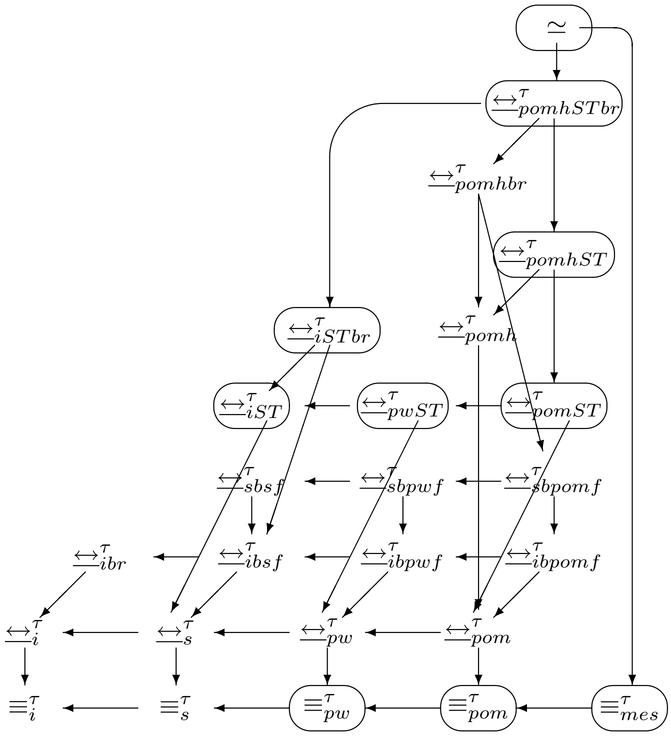

Figure 1: Interrelations of the τ-equivalences and their preservation by SM-refinements for Petri nets with silent transitions

w.r.t. SM-refinements was proved only for $\underline{\leftrightarrow}^{\tau}_{pomhST}$ in [3] and for $\underline{\leftrightarrow}^{\tau}_{iST}$ in [12]. The preservation result for other ST-τ-bisimulation equivalences was proved in [26], but it was done on event structures and an other refinement operator was used. The preservation of trace τ-equivalences was not established before. Thus, our results for $\underline{\leftrightarrow}^{\tau}_{pwST}$, $\underline{\leftrightarrow}^{\tau}_{pomST}$, $\equiv^{\tau}_{pw}$ and $\equiv^{\tau}_{pom}$ are also new.

These results are shown in Figure 1. The interrelations of equivalences are depicted by arrows, and the relations preserved by refinements are placed in ovals.

In addition, we investigated the interrelations of all the τ-equivalence notions on nets without silent transitions and sequential nets. We proved that on nets without silent transitions τ-equivalences coincide with equivalence notions which do not abstract of silent actions. In Figure 2, interrelations of equivalences on nets without silent transitions are presented.

We demonstrated that on sequential nets interleaving and pomset τ-equivalences are merged, and back-forth τ-equivalences coincide with forth τ-equivalence relations. In Figure 3, interrelations of equivalences

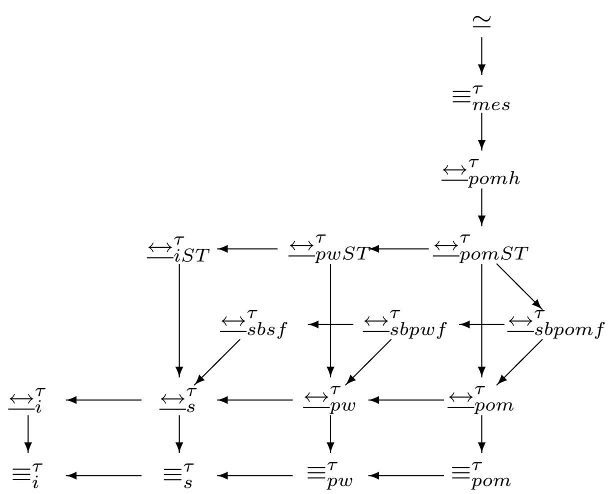

Figure 2: Interrelations of the τ-equivalences on Petri nets without silent transitions

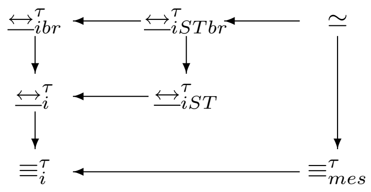

Figure 3: Interrelations of the τ-equivalences on sequential nets

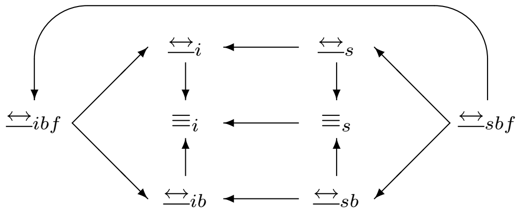

Figure 4: Interrelations of the equivalences for DTSPN's

on sequential nets are shown.

2. We proposed several equivalence relations abstracting of silent actions for the presented class of stochastic nets. The equivalences are defined on the basis of the *observable* state graph of a DTSPN which is a standard state graph after eliminating the arcs corresponding to silent transitions or loops due to a waiting in a state. It was shown that the relations preserve interesting aspects of system behavior. The equivalences can be used to compare different systems and to compute for a system a minimal equivalent representation [5]. The latter aspect is especially interesting for bisimulation equivalences, for which efficient algorithms exist to compute the largest bisimulation for a given net. By representation each equivalence class of this relation by a single marking we obtain a minimal representation at the state transition level.

   Trace relations we defined are: interleaving ($\equiv_i$) and step ($\equiv_s$) equivalences. (Forward) bisimulation relations include interleaving ($\underline{\leftrightarrow}_i$) and step ($\underline{\leftrightarrow}_s$) ones. In addition, notions of backward bisimulation (which respect the "past" of a behaviour) were proposed: interleaving ($\underline{\leftrightarrow}_{ib}$) and step ($\underline{\leftrightarrow}_{sb}$). We presented also back and forth bisimulations (respecting both "past" and "future"): interleaving ($\underline{\leftrightarrow}_{ibf}$) and step ($\underline{\leftrightarrow}_{sbf}$). As a result of comparing the equivalences in accordance to differentiating power, we obtained a lattice of implications presented in Figure 4. Thus, we provided the new variant of stochastic Petri nets with a variety of interleaving and step semantics. The step relations naturally correspond to non-interleaving character of the model. In addition, we demonstrated an application of the equivalences for comparing stationary behavior of DTSPNs.

   Apart from SPNs, stochastic process algebras (SPAs) became very popular as a modeling framework in the recent years. Of particular interest is the relationship between SPNs and SPAs. In [8, 17], an Algebra of Finite nondeterministic Processes AFP_0 was proposed. Its formulas specify a special subclass of Petri nets: Acyclic or A-nets (ANs). There are three basic operations in the algebra: sequential composition, alternative and concurrency. We proposed a stochastic extension of this calculus: algebra $StAFP_0$ describing Stochastic A-nets (SANs), a subclass of DTSPNs. In $StAFP_0$, we use activities instead of actions. An activity is a pair consisting of action and its probability. Activities are basic elements to build formulas of the stochastic calculus.

   For the net equivalence $=_{net}$ (an isomorphism of net representations of algebraic formulas) we presented a sound axiom system Θ_{net}. We prove that any formula of $StAFP_0$ can be transformed (with use of Θ_{net}) into an equivalent (via $=_{net}$) totally stratified one. A totally stratified formula has a form of conjunction (via concurrency) with conjunctive members containing no concurrency operations. Thus, each such member can contain only operations of sequential composition and alternative, and it describes a sequential stochastic process. Hence, we can always find components of a formula corresponding to concurrently composed subnets of a SAN. This makes easier the procedures of analysis and comparison of stochastic processes specified by formulas of $StAFP_0$.

Prepared papers

1. Tarasyuk, I.V. *τ-equivalences and refinement for Petri nets based design. Technische Berichte* **TUD-FI00-11**, 41 pages, Fakultät Informatik, Technische Universität Dresden, Germany, November 2000.

2. Buchholz, P., Tarasyuk, I.V. *A class of stochastic Petri nets with step semantics and related equivalence notions. Technische Berichte* **TUD-FI00-12**, 18 pages, Fakultät Informatik, Technische Universität Dresden, Germany, November 2000.

3. Buchholz, P., Tarasyuk, I.V. *Net and algebraic approaches to probablistic modeling. Joint Novosibirsk Computing Center and Institute of Informatics Systems Bulletin, Series Computer Science* **15**, 35 pages, Novosibirsk, 2001 (to be published).

Participating scientific events and teaching activity

- 1^{st} *Euro Summer School on Trends in Computer Science – 00 (FMPA'00)*, Berg en Dal (Nijmegen), The Netherlands, July 3–7, 1999.

- *Workshop on the Graduate Programme of Technical University of Dresden: Specification of Discrete Processes and Systems of Processes by Operational Models and Logics (SPOML'01)*, Reinhardtsdorf (Schöna), Saxony, Germany, February 5–9, 2001.

- 18^{th} *Annual Symposium on Theoretical Aspects of Computer Science (STACS'01)*, Dresden, Germany, February 15–17, 2001.

- *Delivering lecture course "Equivalence relations for net and algebraic models of concurrency"* at the Department of Computer Science of Technical University, Dresden, Germany, April – July 2001.

- 1^{st} *International School on Formal Methods for the Design of Computer, Software and Communication Systems: Process Algebras (SFM-01:PA)*, Bertinoro, Italy, July 23–28, 2001.

- 13^{th} *European Summer School on Logic, Language and Information (ESSLLII'01)*, Helsinki, Finland, August 13–24, 2001.

- 6^{th} *International Conference on Parallel Computing Technologies - 2001 (PaCT'01)*, Novosibirsk, Russia, September 3–7, 2001.

Future research Possible extension of this work can be an attempt to define other bisimulation equivalences of stochastic Petri nets in interleaving and step semantics. For example, branching bisimulation [23] can be considered as well as variants of back-forth equivalences defined in [21, 22]. For these equivalences we cannot use observable state graphs, since we may need lower level information. For example, to define branching relations, we should respect occurrences of invisible transitions and states where they conflict with other ones. Thus, we cannot just abstract of invisible transitions from very beginning. To propose notions of back-forth bisimulations, we need an information about the path of events which came to the present state. Hence, this is not enough even to consider paths of transitions which led from the initial marking to the present one, since the same transitions can happen concurrently or sequentially resulting the same marking (in non-safe nets). In such a case, we should have something like processes for stochastic nets and collect events for out of paths from such processes. We may also define true concurrent equivalences for stochastic nets such that partial word or pomset ones [23, 25, 26]. Step semantics we proposed can be the first stage to true concurrent semantics for stochastic nets.

We could also enrich our algebraic specifications with an ability to describe infinite processes such as recursion operation. But for this purpose our calculus is too restrictive because of synchronization by names. This means that an action cannot depend on equally named one which is essential for a recursion. A possible decision is to use more flexible calculus as a basic one to enrich with stochastic features. We consider Petri Box Calculus (PBC) [2] as the most appropriate candidate.

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